ANALYTICAL APPROXIMATIONS
IN PROBABILISTIC ANALYSIS OF REAL-TIME SYSTEMS

RTSS 2022
December 7, 2022

Filip Marković, Thomas Nolte, and Alessandro Vittorio Papadopoulos

MAX PLANCK INSTITUTE
FOR SOFTWARE SYSTEMS

HIGH-LEVEL PROBLEM STATEMENT

HIGH-LEVEL PROBLEM STATEMENT

*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*

HIGH-LEVEL PROBLEM STATEMENT

*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*

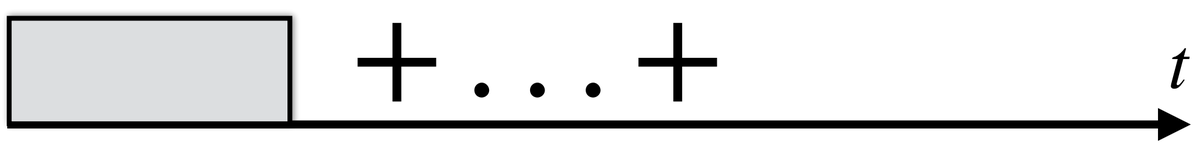
HIGH-LEVEL PROBLEM STATEMENT

*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*

job's execution

HIGH-LEVEL PROBLEM STATEMENT

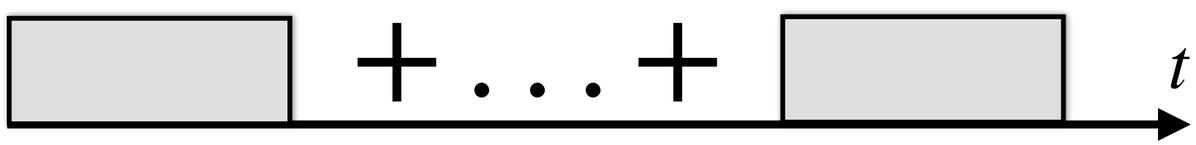
*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*



job's execution

HIGH-LEVEL PROBLEM STATEMENT

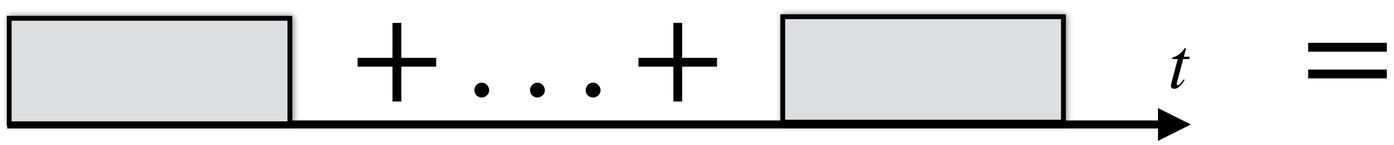
*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*



job's execution *another job's execution*

HIGH-LEVEL PROBLEM STATEMENT

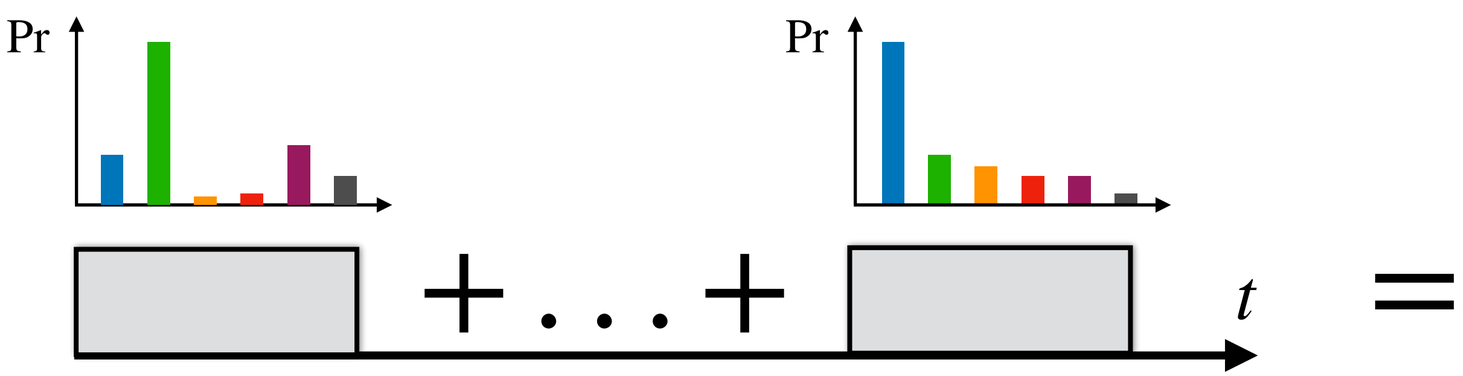
*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*



job's execution *another job's execution*

HIGH-LEVEL PROBLEM STATEMENT

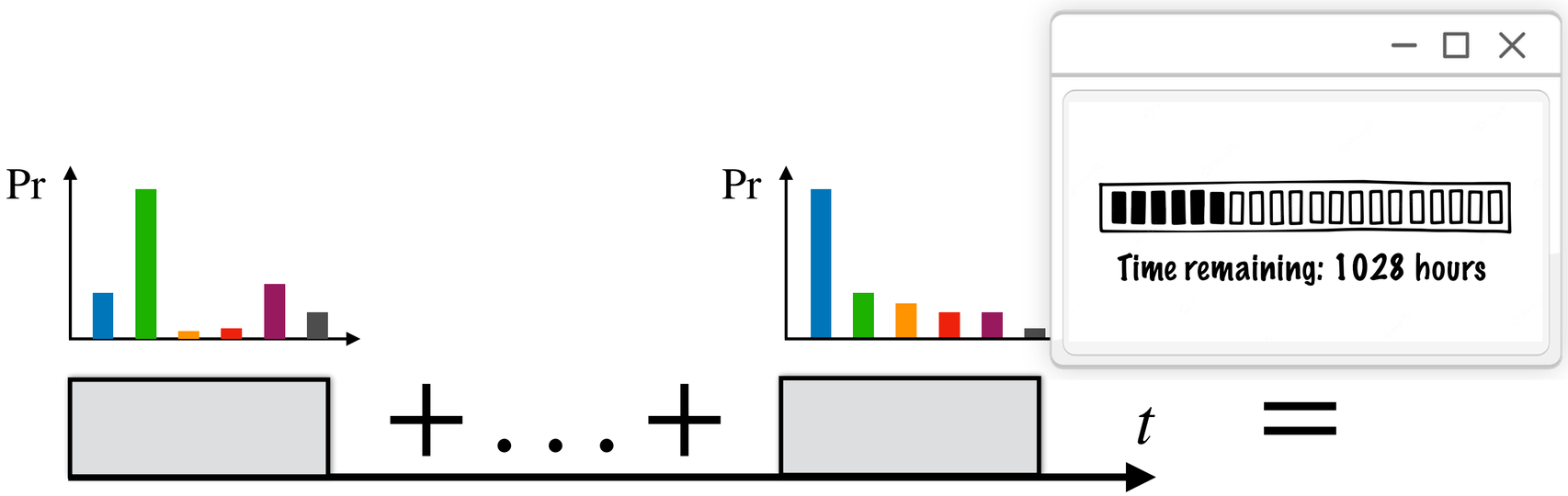
*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*



job's execution *another job's execution*

HIGH-LEVEL PROBLEM STATEMENT

*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*

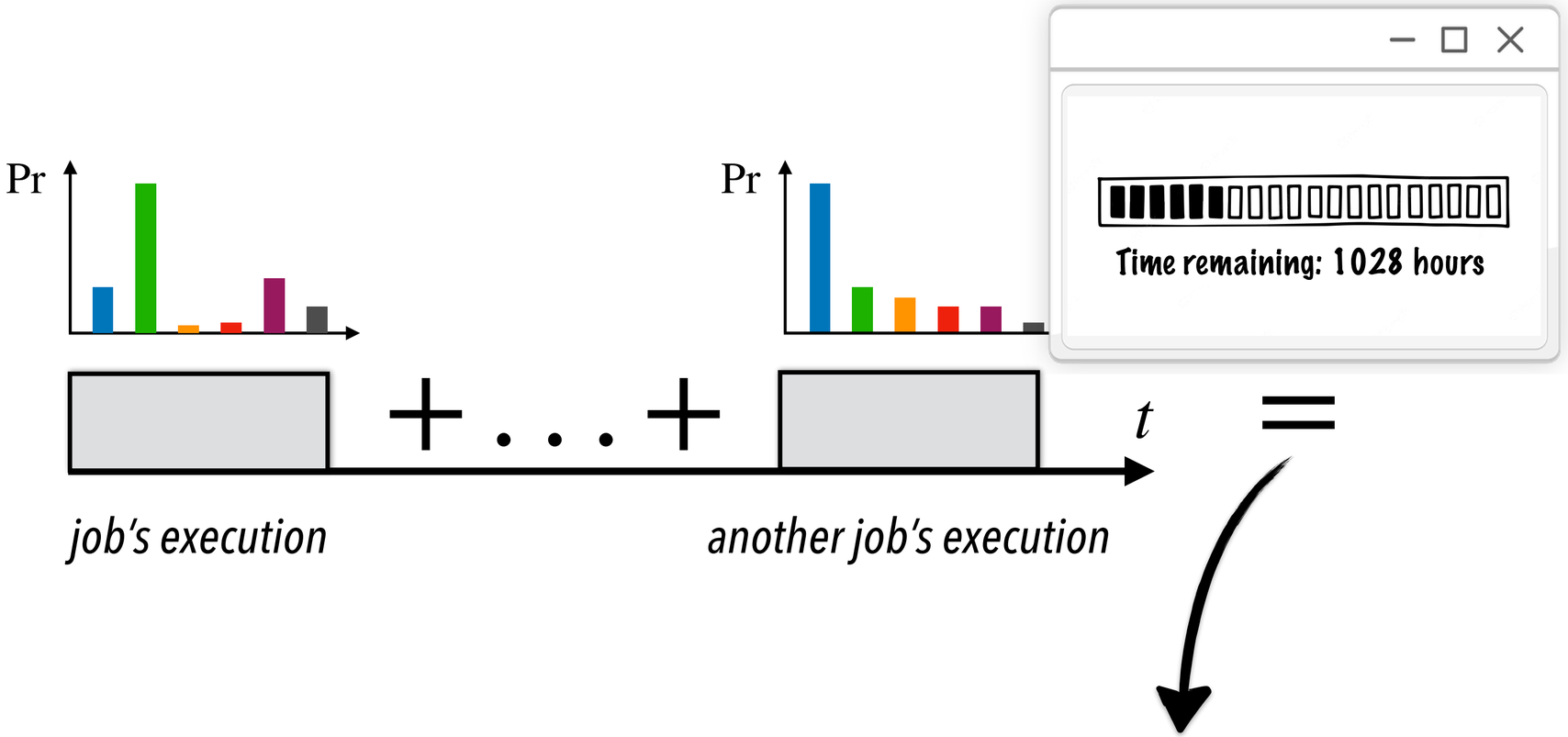


job's execution

another job's execution

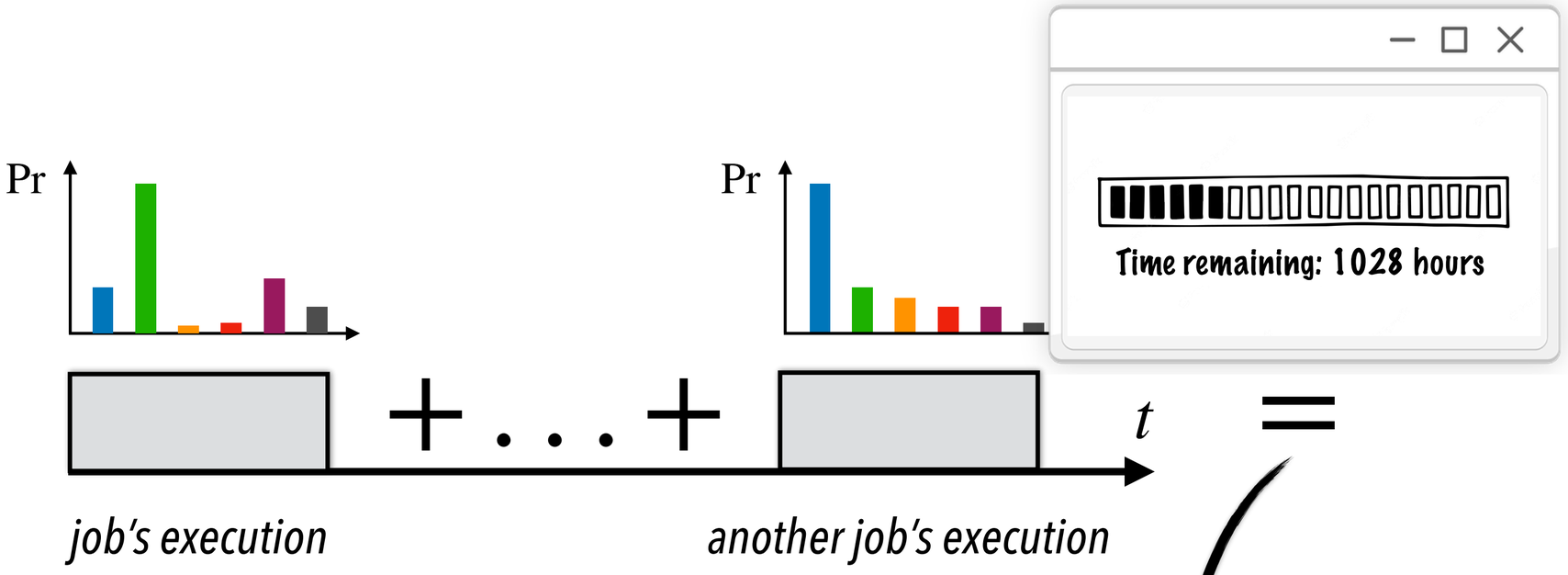
HIGH-LEVEL PROBLEM STATEMENT

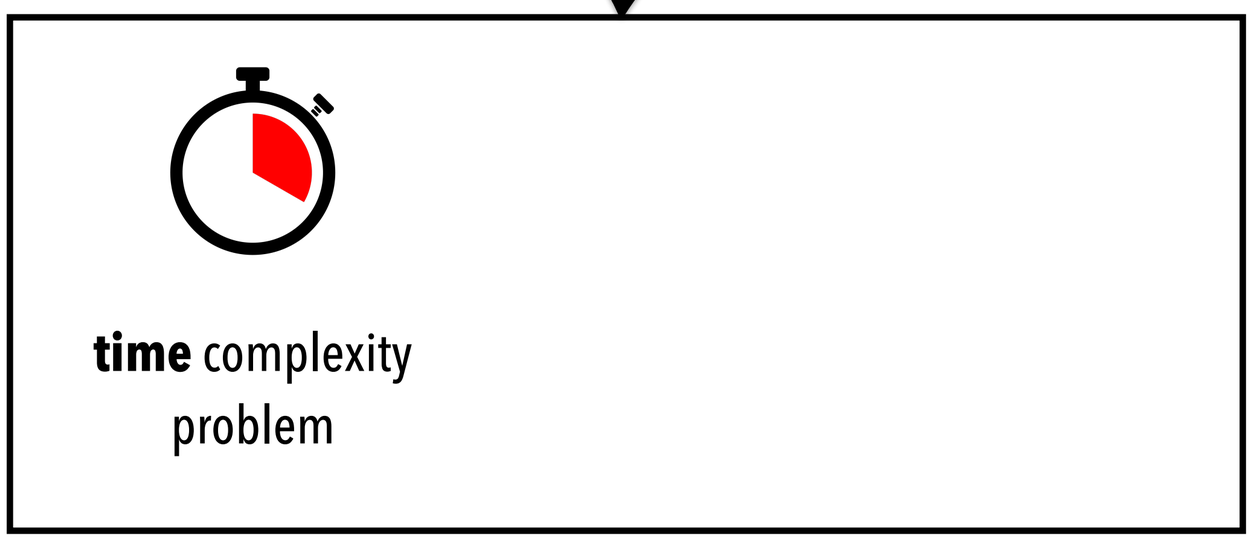
*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*



HIGH-LEVEL PROBLEM STATEMENT

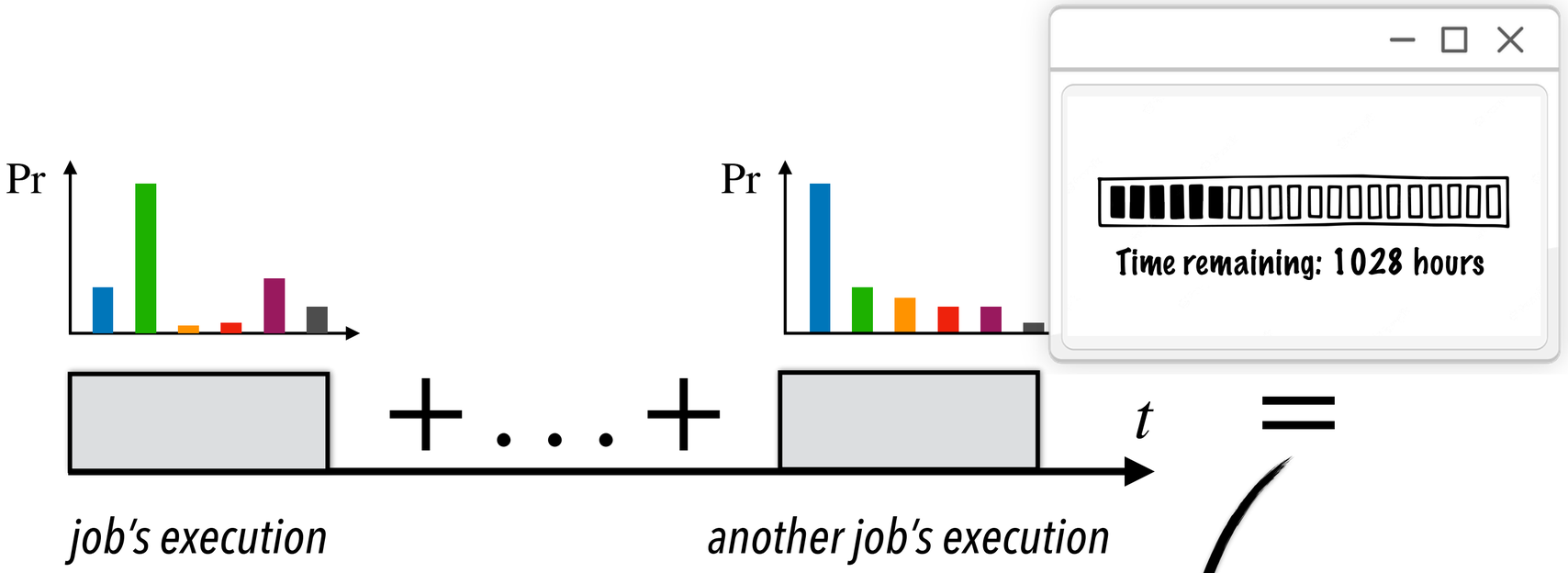
*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*

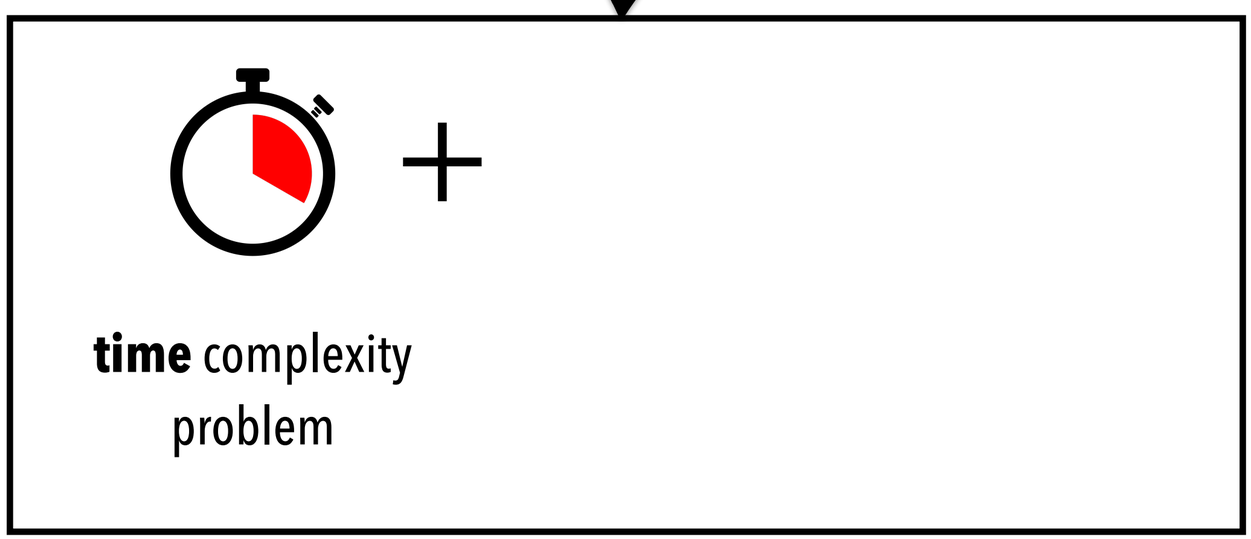




HIGH-LEVEL PROBLEM STATEMENT

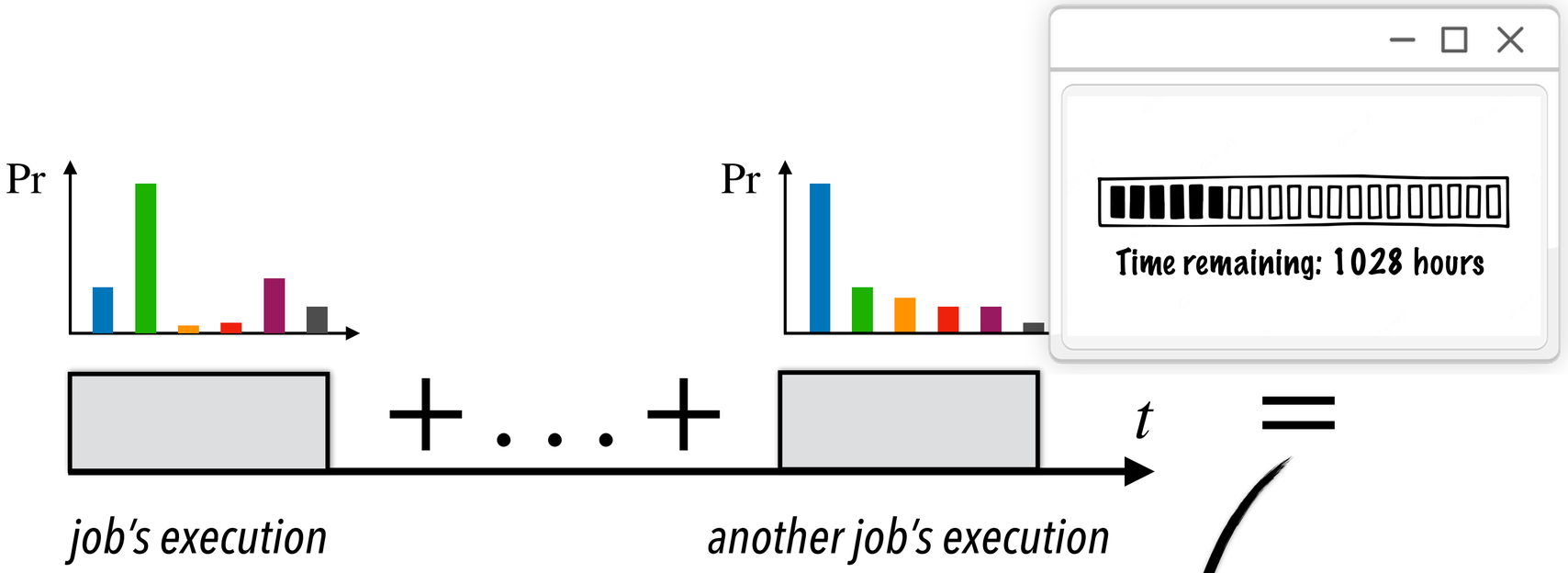
*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*

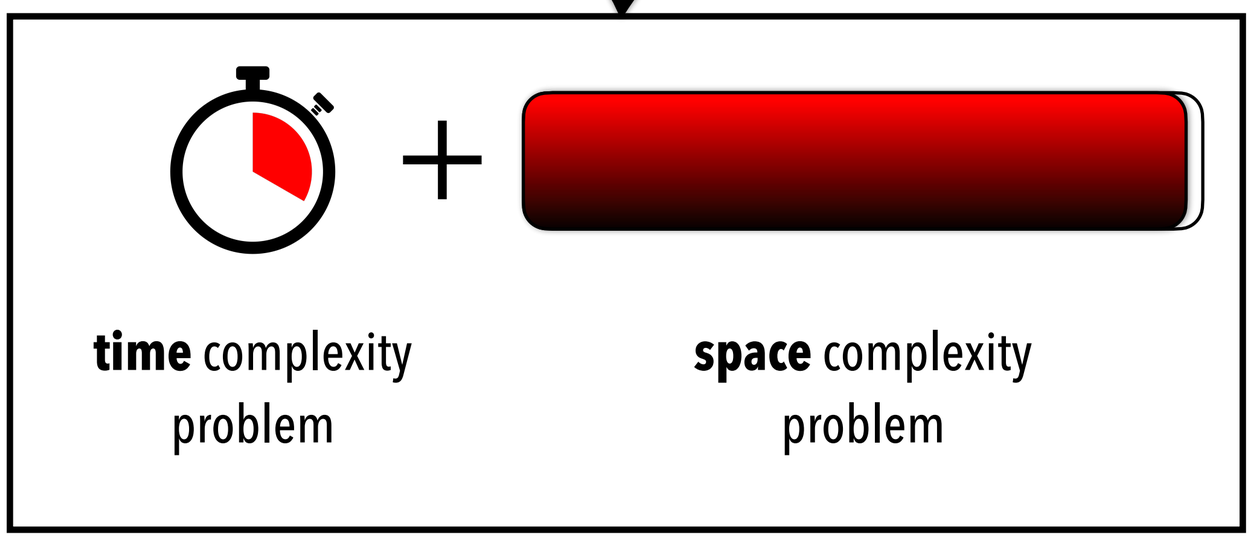




HIGH-LEVEL PROBLEM STATEMENT

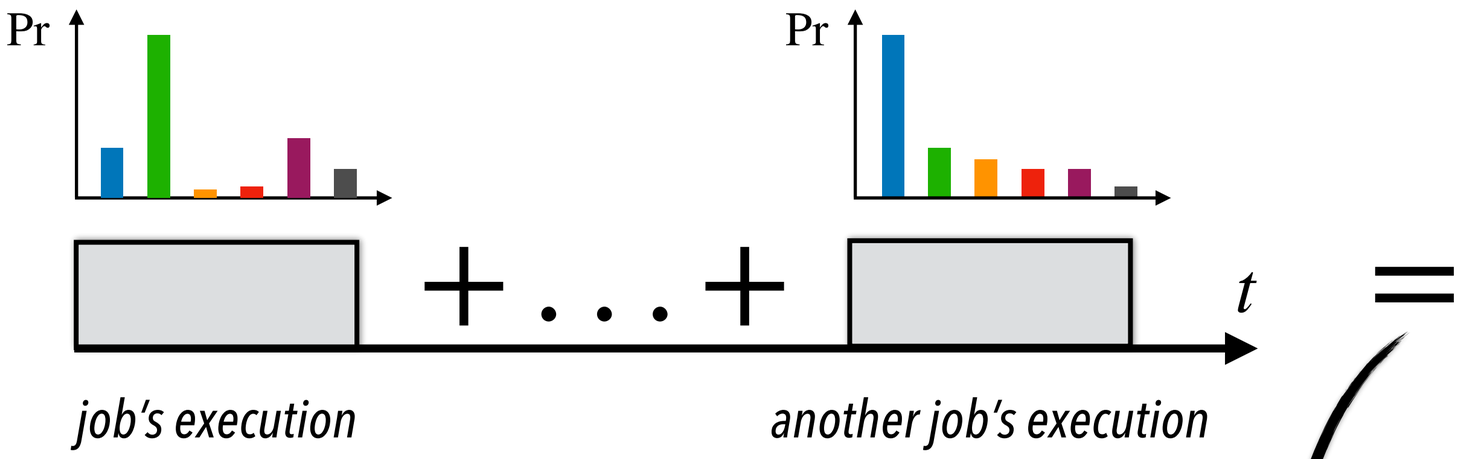
*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*

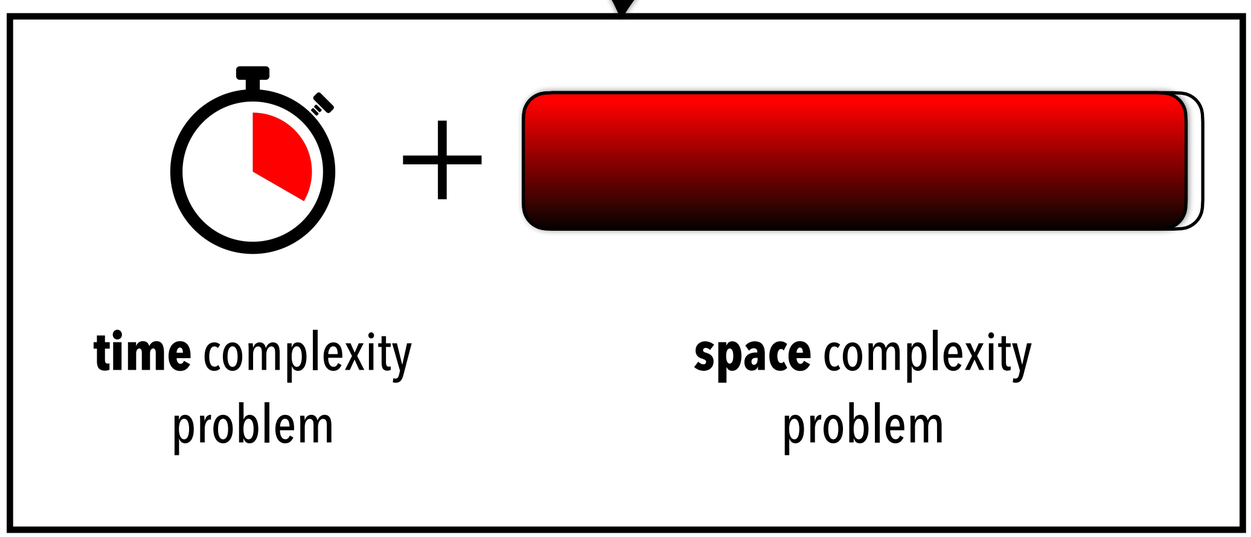




HIGH-LEVEL PROBLEM STATEMENT

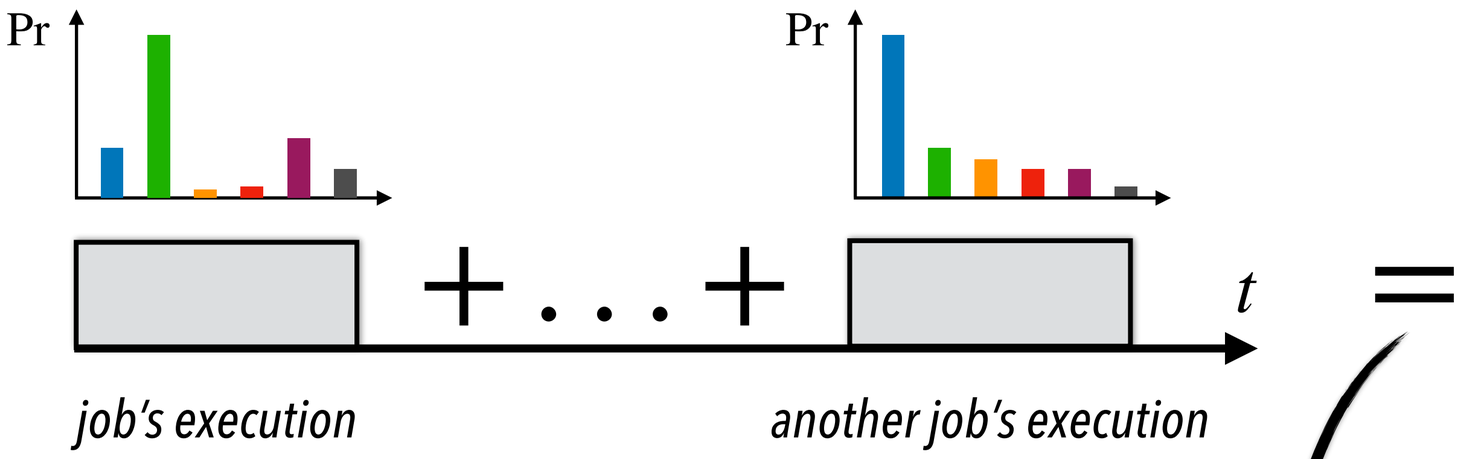
*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*

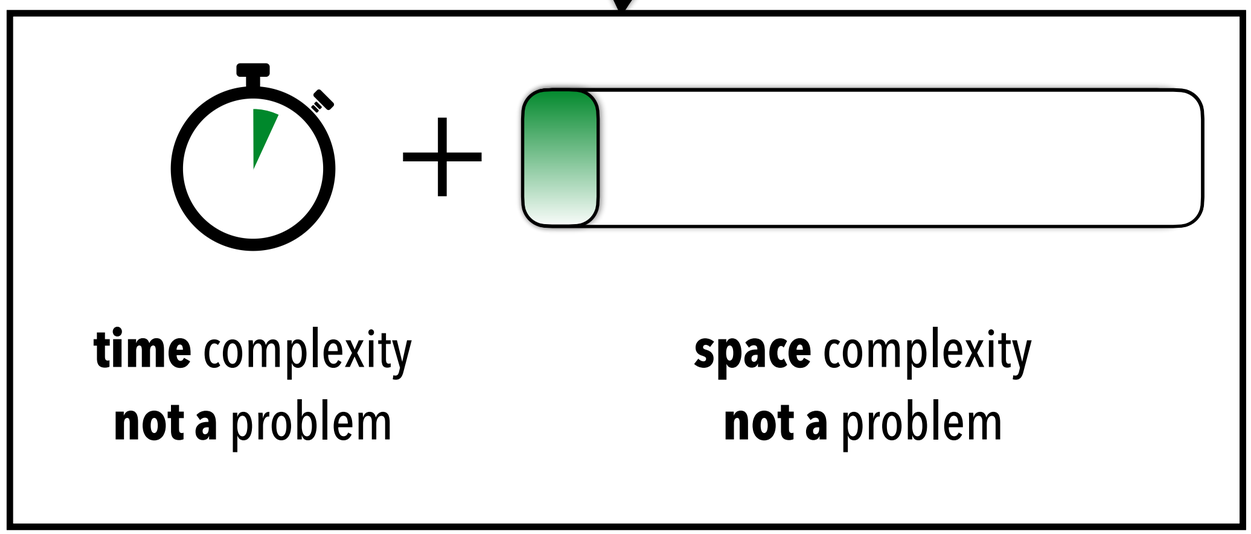




HIGH-LEVEL PROBLEM STATEMENT

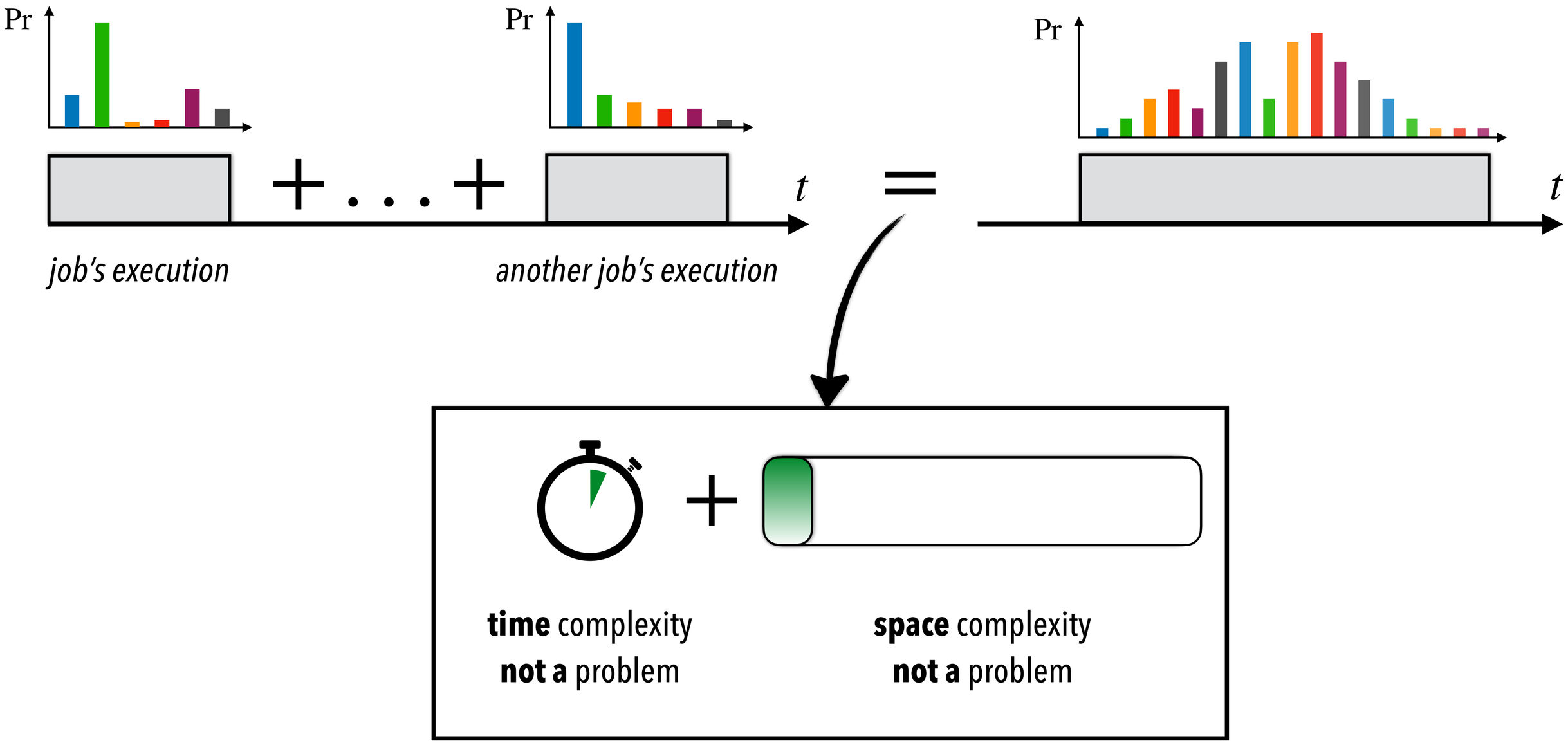
*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*





HIGH-LEVEL PROBLEM STATEMENT

*How to make the probabilistic analysis of RT systems **fast** and **memory efficient**?*



BACKGROUND

BACKGROUND

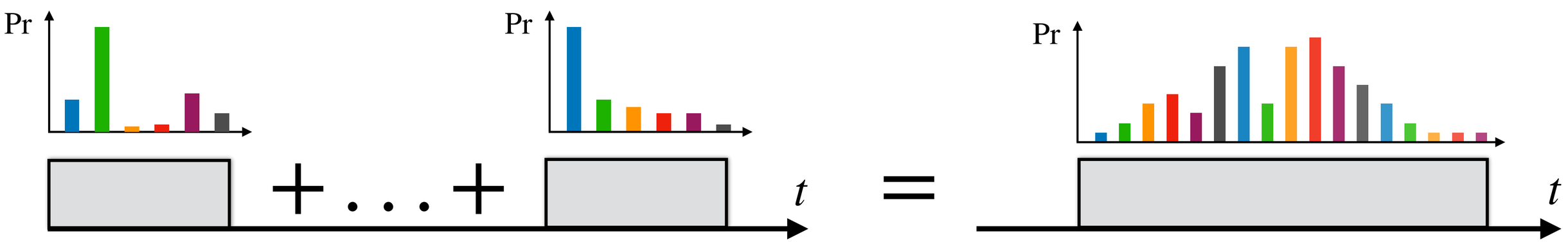
*Computing the execution **distribution** resulting from many interacting job distributions is an **important problem**.*

BACKGROUND

*Computing the execution **distribution** resulting from many interacting job distributions is an **important problem**.*

BACKGROUND

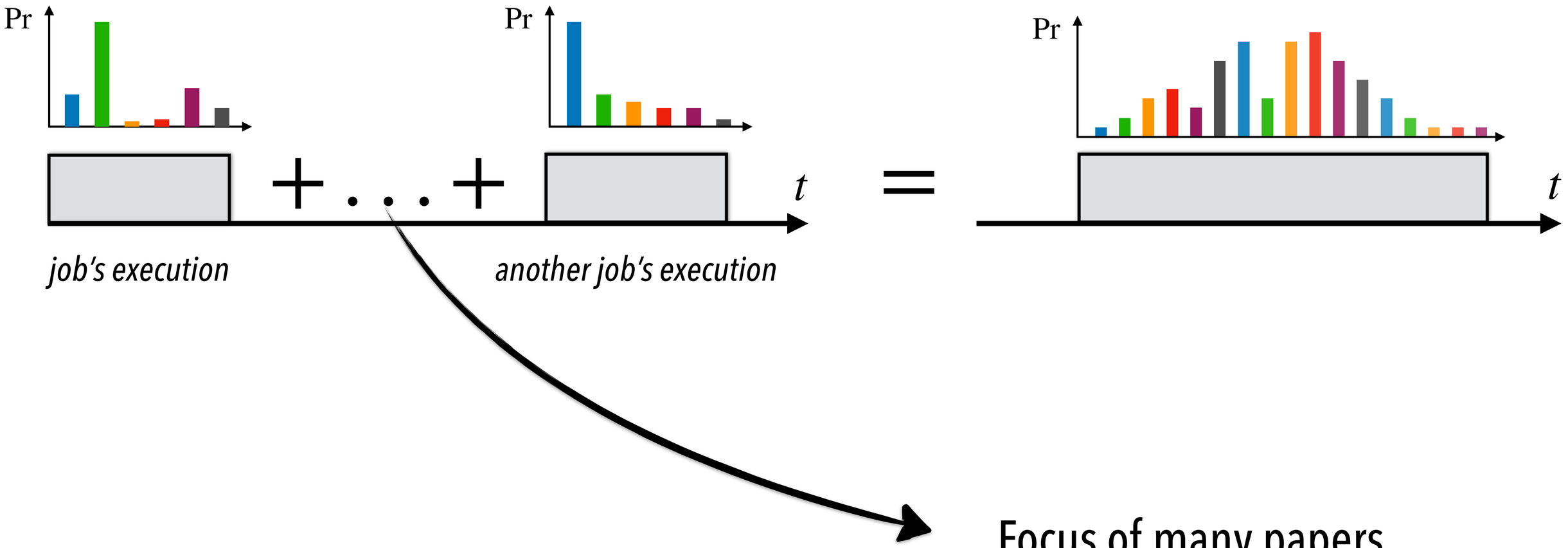
*Computing the execution **distribution** resulting from many interacting job distributions is an **important problem**.*



job's execution *another job's execution*

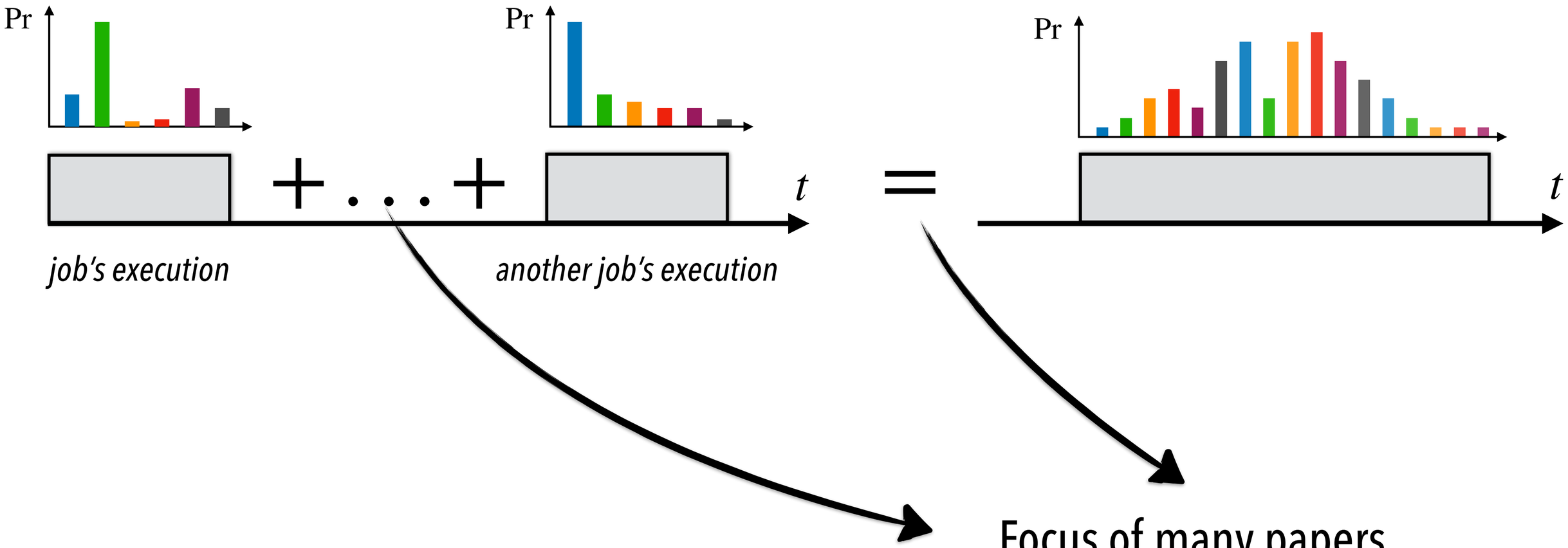
BACKGROUND

*Computing the execution **distribution** resulting from many interacting job distributions is an **important problem**.*



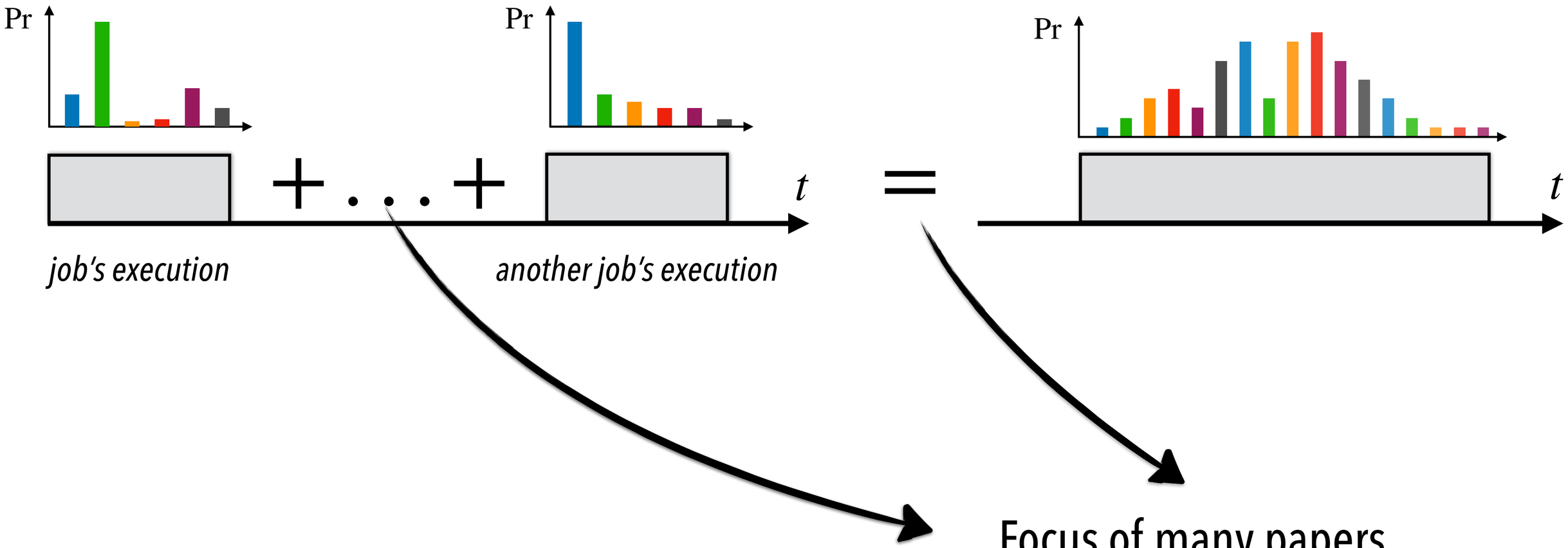
BACKGROUND

*Computing the execution **distribution** resulting from many interacting job distributions is an **important problem**.*



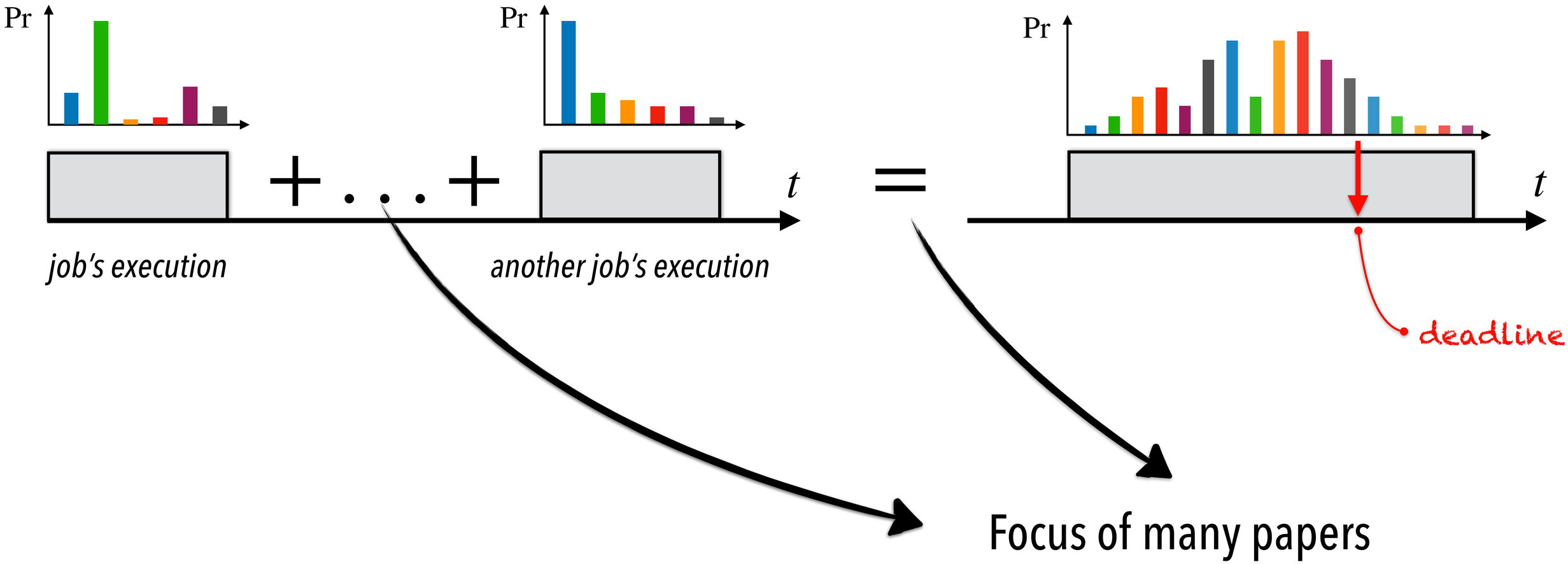
BACKGROUND

*Ultimately, the resulting distribution provides important information on the **deadline-miss probability**.*



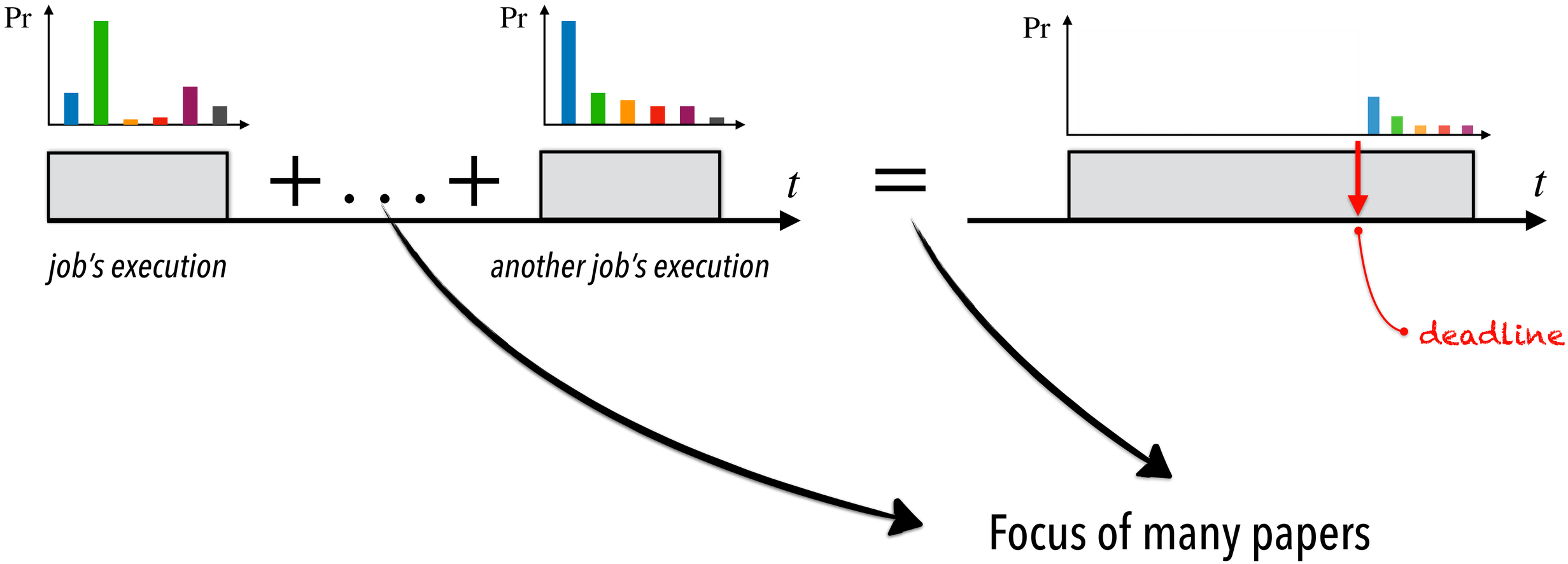
BACKGROUND

*Ultimately, the resulting distribution provides important information on the **deadline-miss probability**.*



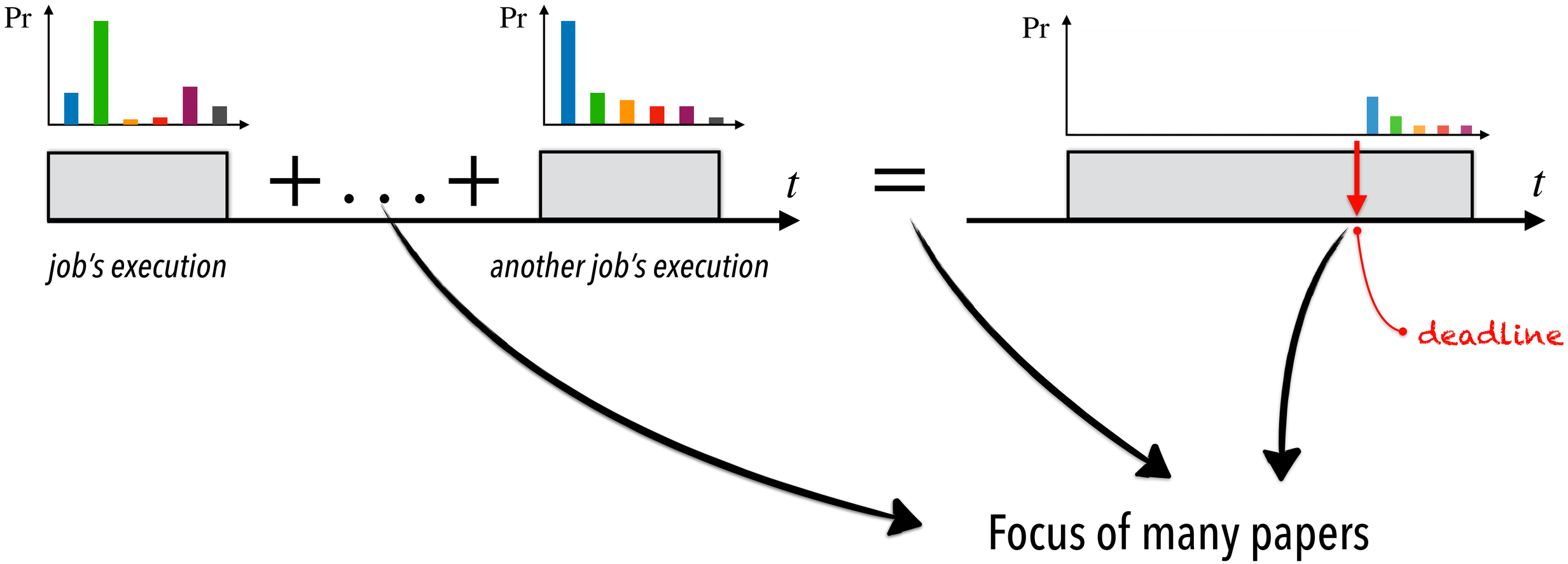
BACKGROUND

*Ultimately, the resulting distribution provides important information on the **deadline-miss probability**.*



BACKGROUND

*Ultimately, the resulting distribution provides important information on the **deadline-miss probability**.*



TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

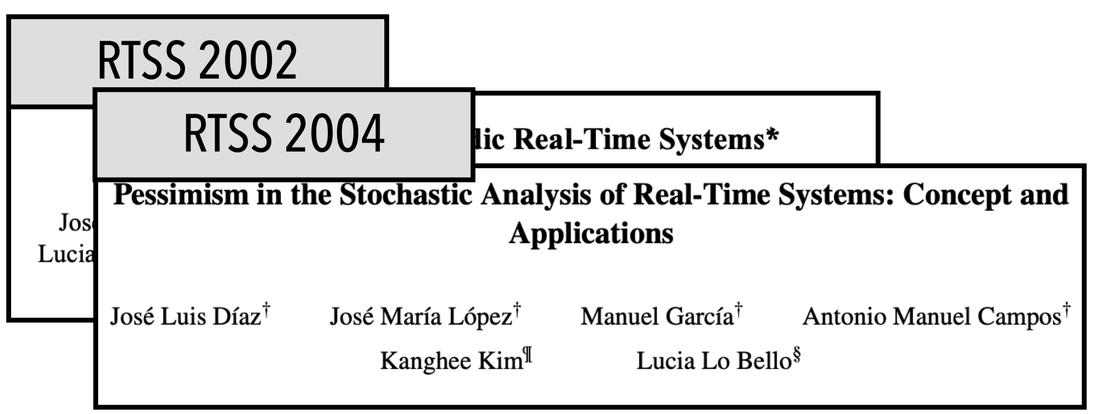
RTSS 2002

Stochastic Analysis of Periodic Real-Time Systems*

José Luis Díaz† Daniel F. García† Kanghee Kim‡ Chang-Gun Lee¶
Lucia Lo Bello§ José María López† Sang Lyul Min‡ Orazio Mirabella§

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

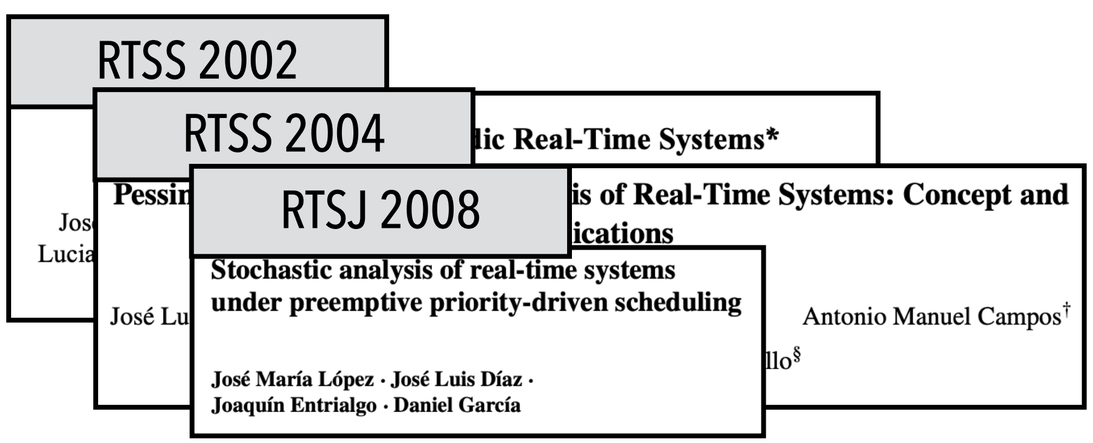


…

t

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

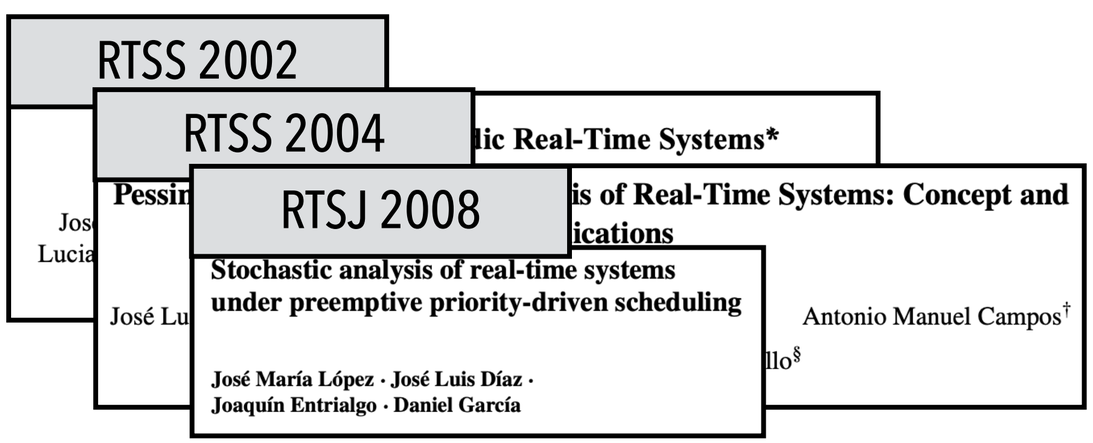


…

t

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*



TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

RTSS 2002

RTSS 2004

RTSJ 2008

Stochastic analysis of real-time systems under preemptive priority-driven scheduling

José María López · José Luis Díaz · Joaquín Entrialgo · Daniel García

RTSS 2013

Response Time Analysis for Fixed-Priority Tasks with Multiple Probabilistic Parameters

Dorin Maxim[1,2,3], Liliana Cucu-Grosjean[1,2,3]
[1] Universite de Lorraine, LORIA, UMR 7503, F-54506, France
[2] CNRS, LORIA UMR 7503, Vandoeuvre-les-Nancy, F54519, France
[3] INRIA Nancy-Grand Est, Villers-les-Nancy, F-54600, France
dorin.maxim@inria.fr, liliana.cucu@inria.fr

… … t

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

RTSS 2002

RTSS 2004

RTSJ 2008

Stochastic analysis of real-time systems under preemptive priority-driven scheduling

José María López · José Luis Díaz · Joaquín Entrialgo · Daniel García

RTSS 2013

Response Time Analysis for Fixed-Priority Tasks with Multiple Probabilistic Parameters

Dorin Maxim[1,2,3], Liliana Cucu-Grosjean[1,2,3]
[1] Universite de Lorraine, LORIA, UMR 7503, F-54506, France
[2] CNRS, LORIA UMR 7503, Vandoeuvre-les-Nancy, F54519, France
[3] INRIA Nancy-Grand Est, Villers-les-Nancy, F-54600, France
dorin.maxim@inria.fr, liliana.cucu@inria.fr

...

...

t

Probabilistic Schedulability Tests for Uniprocessor Fixed-Priority Scheduling under Soft Errors

Kuan-Hsun Chen and Jian-Jia Chen
Department of Informatics, TU Dortmund University, Germany
{kuan-hsun.chen, jian-jia.chen}@tu-dortmund.de

SIES 2017

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

RTSS 2002

RTSS 2004

RTSJ 2008

Stochastic analysis of real-time systems under preemptive priority-driven scheduling

José María López · José Luis Díaz · Joaquín Entrialgo · Daniel García

RTSS 2013

Response Time Analysis for Fixed-Priority Tasks with Multiple Probabilistic Parameters

Dorin Maxim[1,2,3], Liliana Cucu-Grosjean[1,2,3]
[1] Universite de Lorraine, LORIA, UMR 7503, F-54506, France
[2] CNRS, LORIA UMR 7503, Vandoeuvre-les-Nancy, F54519, France
[3] INRIA Nancy-Grand Est, Villers-les-Nancy, F-54600, France
dorin.maxim@inria.fr, liliana.cucu@inria.fr

ECRTS 2018

Efficiently Approximating the Probability of Deadline Misses in Real-Time Systems

Georg von der Brüggen
Department of Computer Science, TU Dortmund University, Germany
georg.von-der-brueggen@tu-dortmund.de
https://orcid.org/0000-0002-8137-3612

Nico Piatkowski
Department of Computer Science, TU Dortmund University, Germany
nico.piatkowski@tu-dortmund.de
https://orcid.org/0000-0002-6334-8042

Kuan-Hsun Chen
Department of Computer Science, TU Dortmund University, Germany
kuan-hsun.chen@tu-dortmund.de
https://orcid.org/0000-0002-7110-921X

Jian-Jia Chen
Department of Computer Science, TU Dortmund University, Germany
jian-jia.chen@cs.uni-dortmund.de
https://orcid.org/0000-0001-8114-9760

Katharina Morik
Department of Computer Science, TU Dortmund University, Germany
katharina.morik@tu-dortmund.de

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

RTSS 2002

RTSS 2004

RTSJ 2008

Stochastic analysis of real-time systems under preemptive priority-driven scheduling

José María López · José Luis Díaz · Joaquín Entrialgo · Daniel García

RTSS 2013

Response Time Analysis for Fixed-Priority Tasks with Multiple Probabilistic Parameters

Dorin Maxim[1,2,3], Liliana Cucu-Grosjean[1,2,3]
[1] Universite de Lorraine, LORIA, UMR 7503, F-54506, France
[2] CNRS, LORIA UMR 7503, Vandoeuvre-les-Nancy, F54519, France
[3] INRIA Nancy-Grand Est, Villers-les-Nancy, F-54600, France
dorin.maxim@inria.fr, liliana.cucu@inria.fr

ECRTS 2018

ECRTS 2021

On the Convolution Efficiency for Probabilistic Analysis of Real-Time Systems

Filip Marković
Mälardalen University, Västerås, Sweden

Alessandro Vittorio Papadopoulos
Mälardalen University, Västerås, Sweden

Thomas Nolte

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

RTSS 2002

RTSS 2004

RTSJ 2008

Stochastic analysis of real-time systems under preemptive priority-driven scheduling

José María López · José Luis Díaz · Joaquín Entrialgo · Daniel García

RTSS 2013

Response Time Analysis for Fixed-Priority Tasks with Multiple Probabilistic Parameters

Dorin Maxim[1,2,3], Liliana Cucu-Grosjean[1,2,3]
[1] Universite de Lorraine, LORIA, UMR 7503, F-54506, France
[2] CNRS, LORIA UMR 7503, Vandoeuvre-les-Nancy, F54519, France
[3] INRIA Nancy-Grand Est, Villers-les-Nancy, F-54600, France
dorin.maxim@inria.fr, liliana.cucu@inria.fr

RTSS 2021

Monte Carlo Response-Time Analysis

Sergey Bozhko, Georg von der Brüggen, and Björn B. Brandenburg
Max Planck Institute for Software Systems (MPI-SWS)

ECRTS 2021

Mälardalen University, Västerås, Sweden

ECRTS 2018

Department of Computer Science, TU Dortmund University, Germany
kuan-hsun.chen@tu-dortmund.de
https://orcid.org/0000-0002-7110-921X

Jian-Jia Chen
Department of Computer Science, TU Dortmund University, Germany
jian-jia.chen@cs.uni-dortmund.de
https://orcid.org/0000-0001-8114-9760

Katharina Morik
Department of Computer Science, TU Dortmund University, Germany
katharina.morik@tu-dortmund.de

…

…

t

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

RTSS 2002

RTSS 2004

RTSJ 2008

Stochastic analysis of real-time systems under preemptive priority-driven scheduling

José María López · José Luis Díaz · Joaquín Entrialgo · Daniel García

RTSS 2013

Response Time Analysis for Fixed-Priority Tasks with Multiple Probabilistic Parameters

Dorin Maxim[1,2,3], Liliana Cucu-Grosjean[1,2,3]
[1] Universite de Lorraine, LORIA, UMR 7503, F-54506, France
[2] CNRS, LORIA UMR 7503, Vandoeuvre-les-Nancy, F54519, France
[3] INRIA Nancy-Grand Est, Villers-les-Nancy, F-54600, France
dorin.maxim@inria.fr, liliana.cucu@inria.fr

…

…

t

RTSS 2022

Critical Instant for Probabilistic Timing Guarantees: Refuted and Revisited

Kuan-Hsun Chen*‡, Mario Günzel§‡, Georg von der Brüggen§, and Jian-Jia Chen§
*University of Twente, The Netherlands, k.h.chen@utwente.nl
, georg.von-der-brueggen, jian-jia.chen}@tu-dortmund.de
contributed equally

ECRTS 2021

Thomas Nolte
Mälardalen University, Västerås, Sweden

ECRTS 2018

Department of Computer Science, TU Dortmund University, Germany
kuan-hsun.chen@tu-dortmund.de
https://orcid.org/0000-0002-7110-921X

Jian-Jia Chen
Department of Computer Science, TU Dortmund University, Germany
jian-jia.chen@cs.uni-dortmund.de
https://orcid.org/0000-0001-8114-9760

Katharina Morik
Department of Computer Science, TU Dortmund University, Germany
katharina.morik@tu-dortmund.de

TIMELINE OF CONTRIBUTIONS

*Our research community has investigated the topic for **more than two decades**.*

RTSS 2002

RTSS 2004

RTSJ 2008

Stochastic analysis of real-time systems under preemptive priority-driven scheduling

José María López · José Luis Díaz · Joaquín Entrialgo · Daniel García

RTSS 2013

Response Time Analysis for Fixed-Priority Tasks with Multiple Probabilistic Parameters

Dorin Maxim[1,2,3], Liliana Cucu-Grosjean[1,2,3]
[1] Universite de Lorraine, LORIA, UMR 7503, F-54506, France
[2] CNRS, LORIA UMR 7503, Vandoeuvre-les-Nancy, F54519, France
[3] INRIA Nancy-Grand Est, Villers-les-Nancy, F-54600, France
dorin.maxim@inria.fr, liliana.cucu@inria.fr

…

…

t

RTSS 2022

Analytical Approximations in Probabilistic Analysis of Real-Time Systems

Filip Marković[1,2], Thomas Nolte[1], and Alessandro Vittorio Papadopoulos[1]

ECRTS 2021

Mälardalen University, Västerås, Sweden

Thomas Nolte

ECRTS 2018

Kuan-Hsun Chen
Department of Computer Science, TU Dortmund University, Germany
kuan-hsun.chen@tu-dortmund.de
https://orcid.org/0000-0002-7110-921X

Jian-Jia Chen
Department of Computer Science, TU Dortmund University, Germany
jian-jia.chen@cs.uni-dortmund.de
https://orcid.org/0000-0001-8114-9760

Katharina Morik
Department of Computer Science, TU Dortmund University, Germany
katharina.morik@tu-dortmund.de

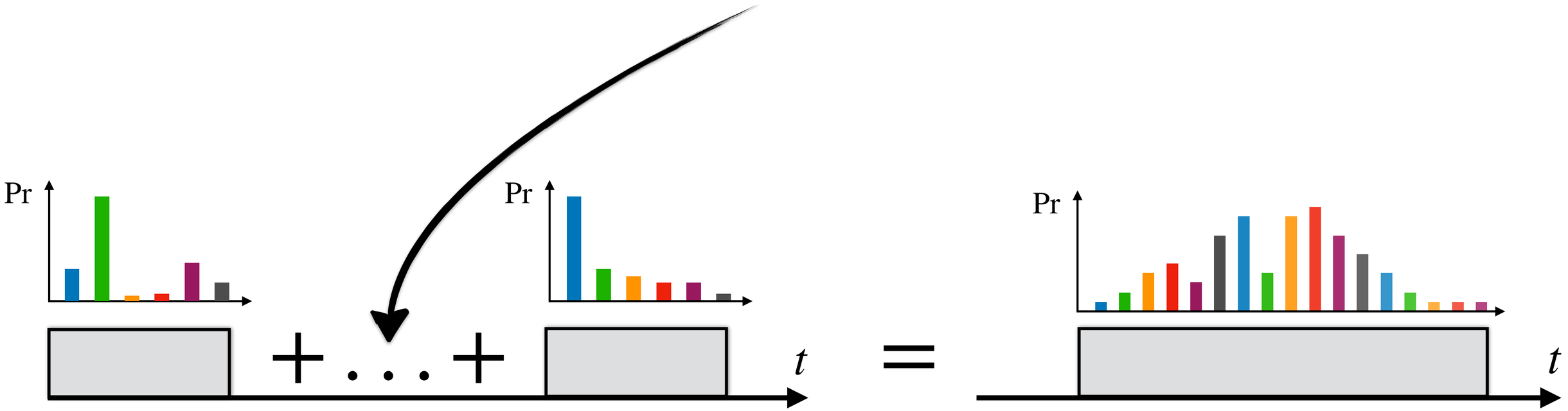
INTUITION BEHIND THE SOLUTION

INTUITION BEHIND THE SOLUTION

*What is the **new insight** from the paper, and **how does it solve efficiency problems**?*

INTUITION BEHIND THE SOLUTION

*Let us first concentrate on the **sum** and investigate how it **behaves**.*



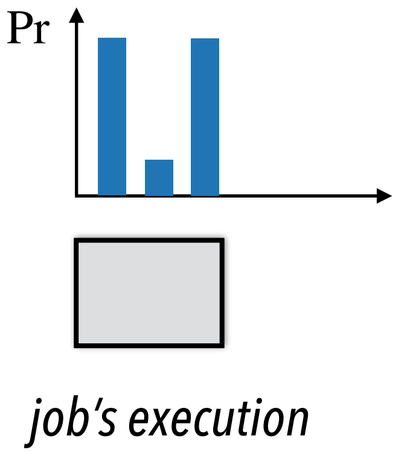
job's execution *another job's execution*

INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*

INTUITION BEHIND THE SOLUTION

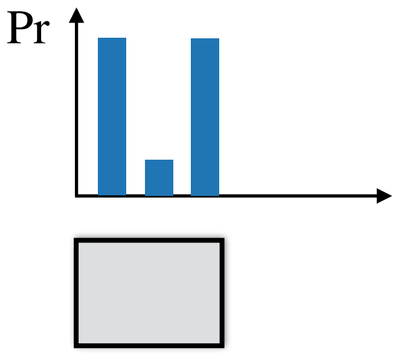
*Consider summing up the following job distribution **several times**.*



job's execution

INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*

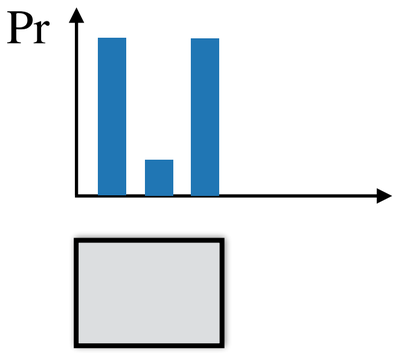


job's execution

=

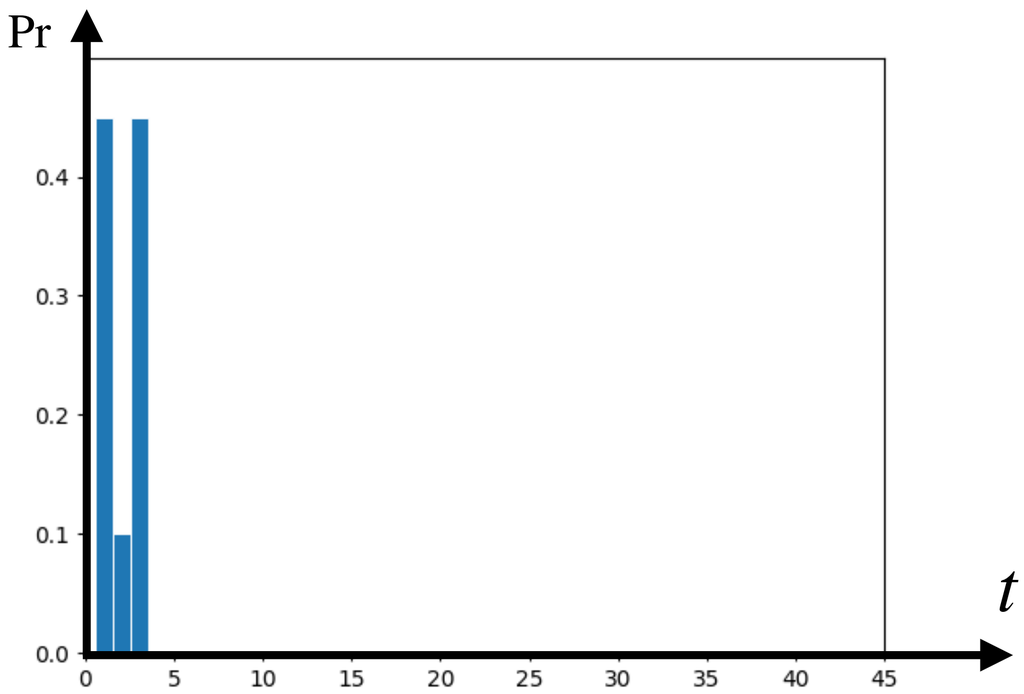
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



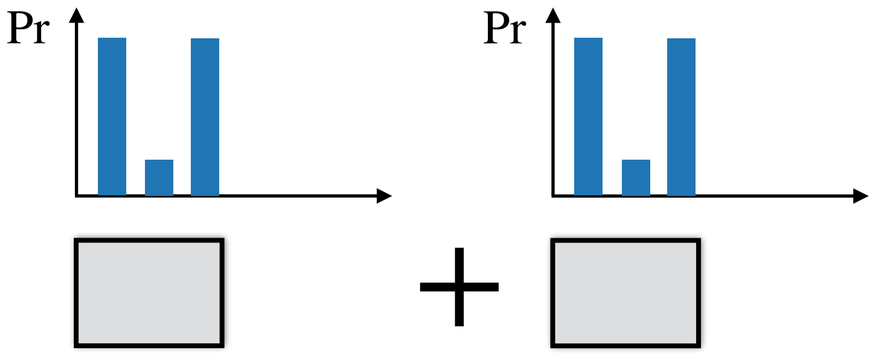
job's execution

=



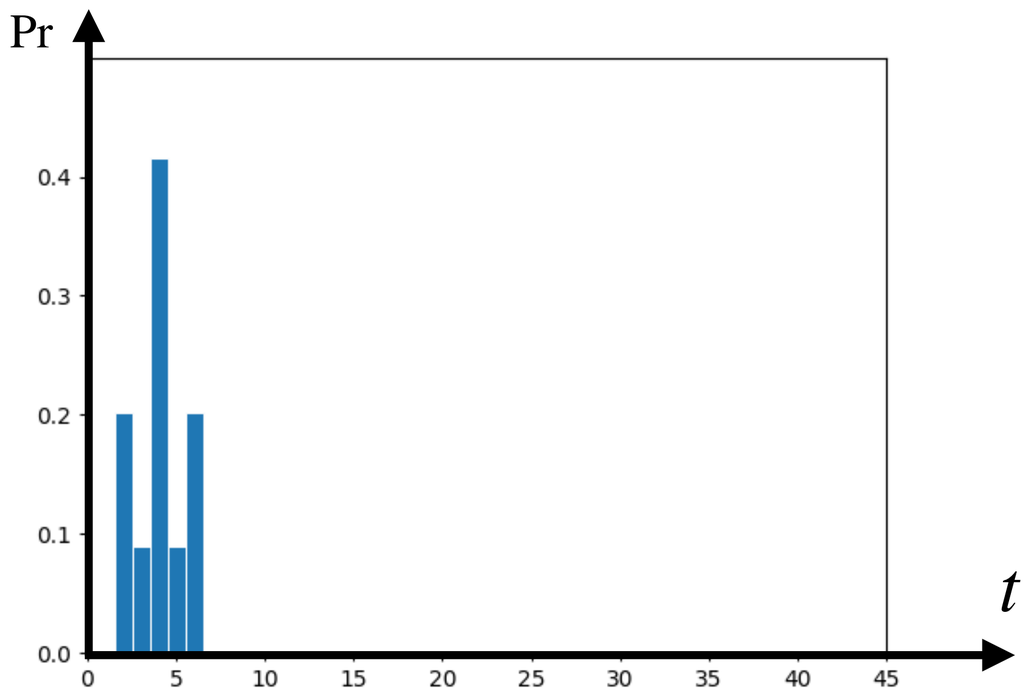
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



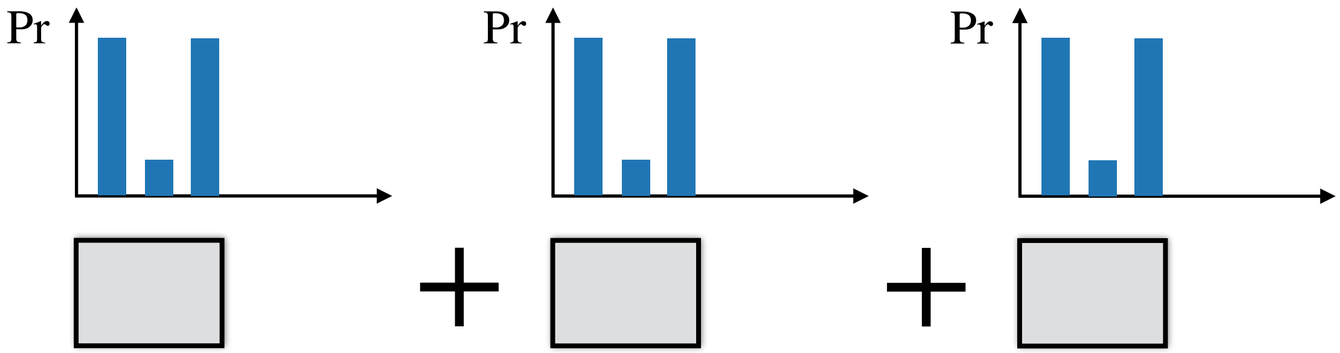
job's execution

=



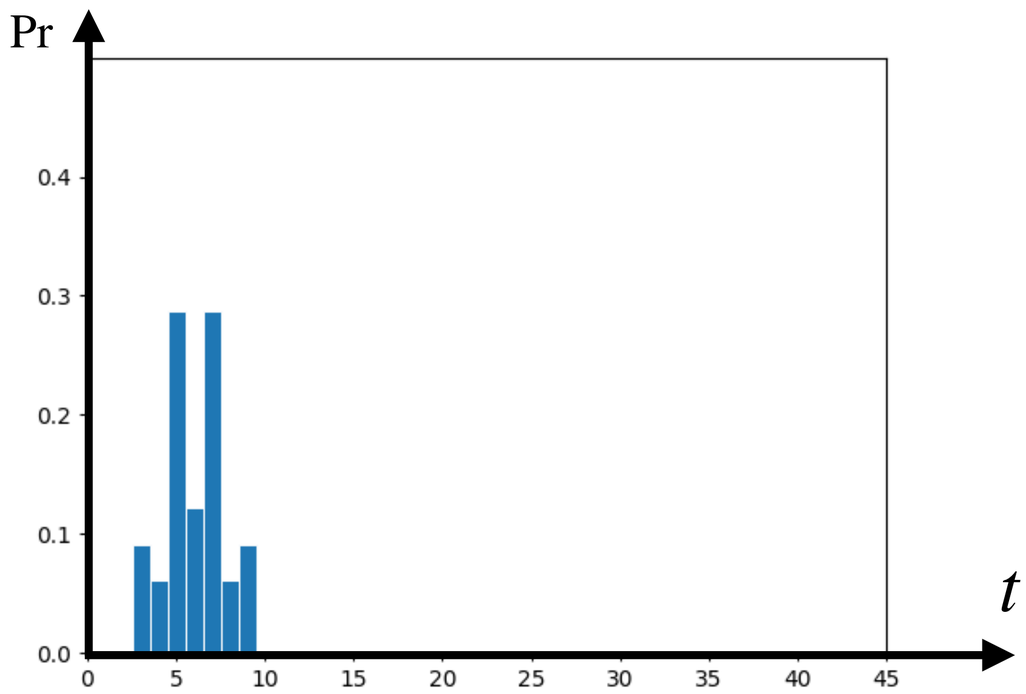
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



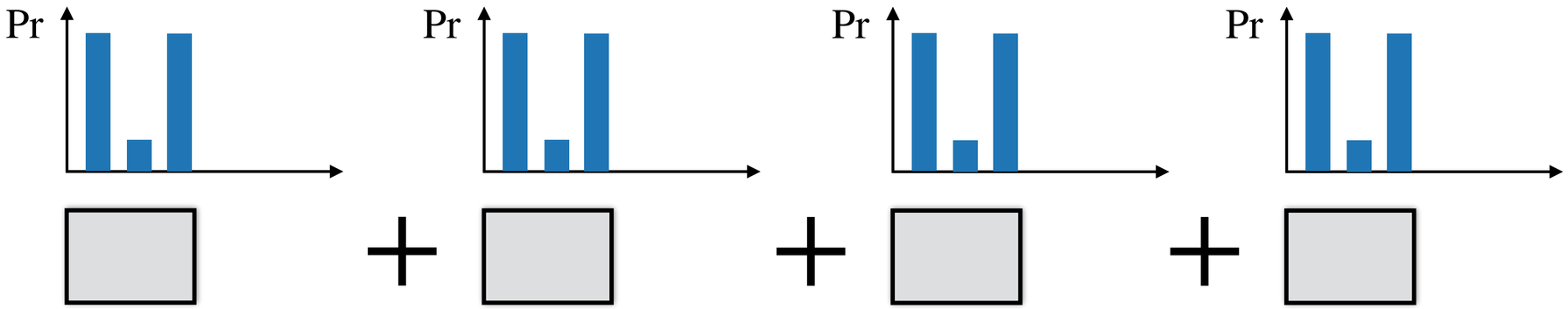
job's execution

=



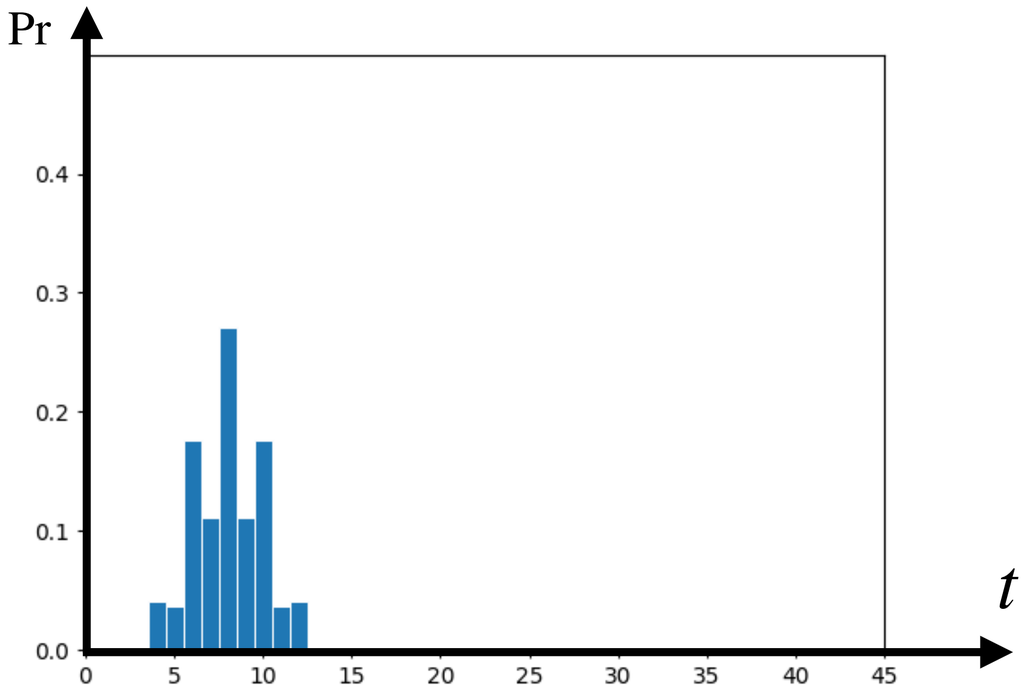
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



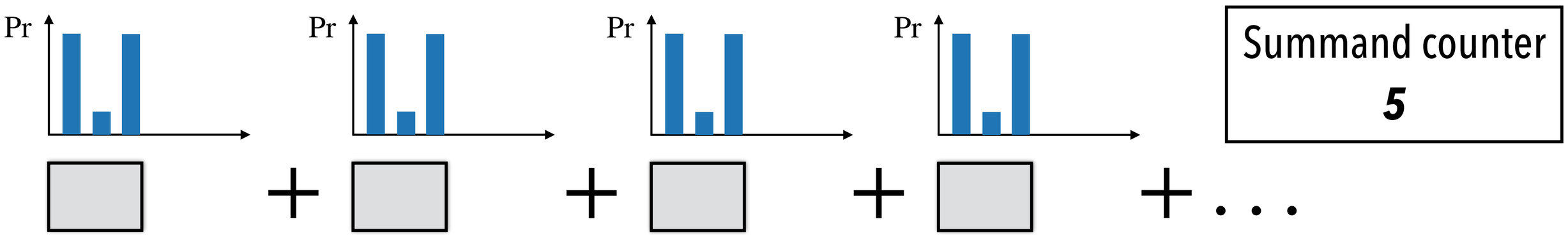
job's execution

=



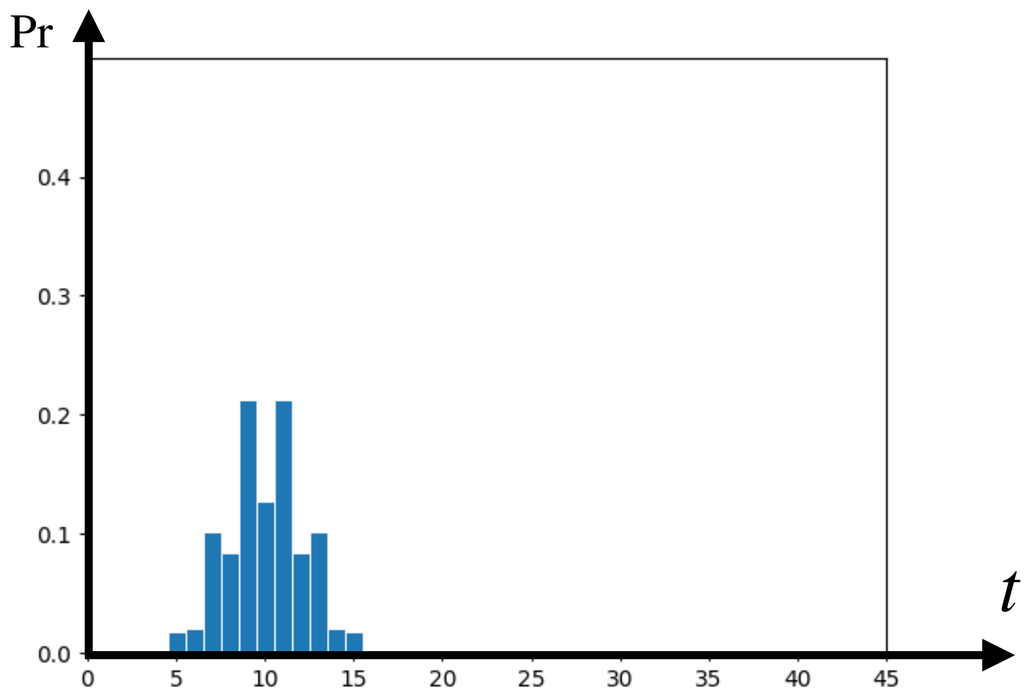
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



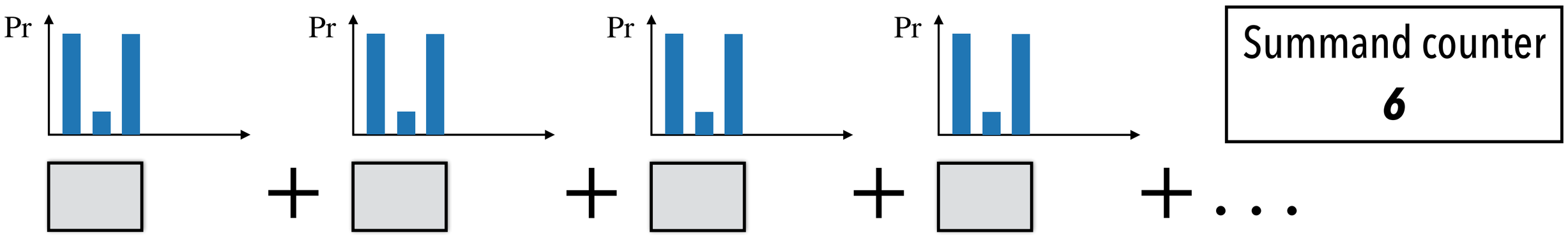
job's execution

=



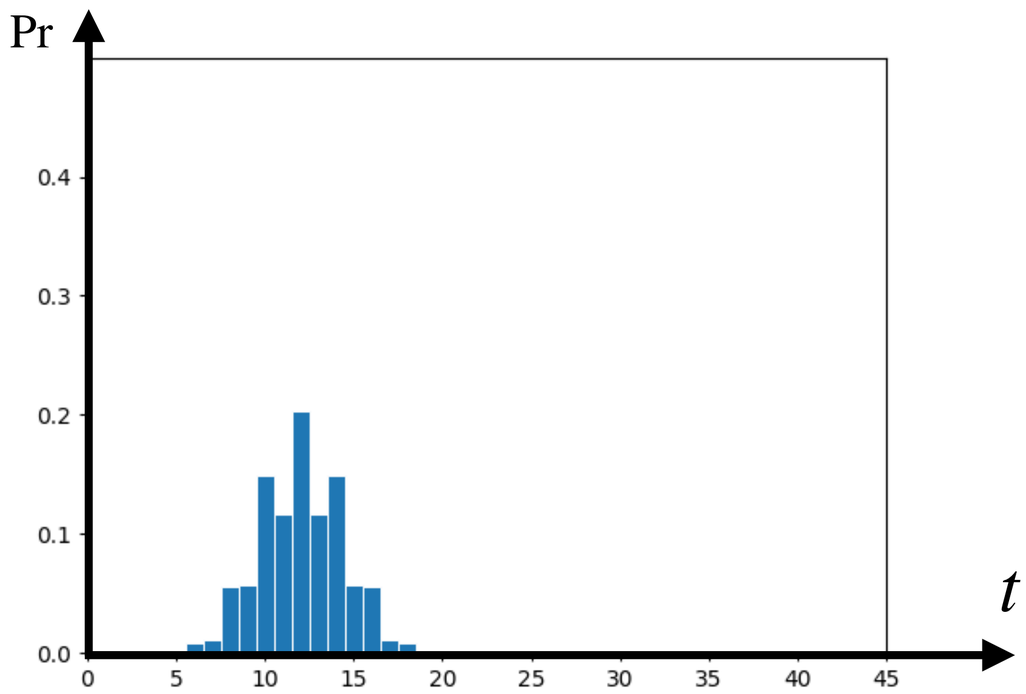
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



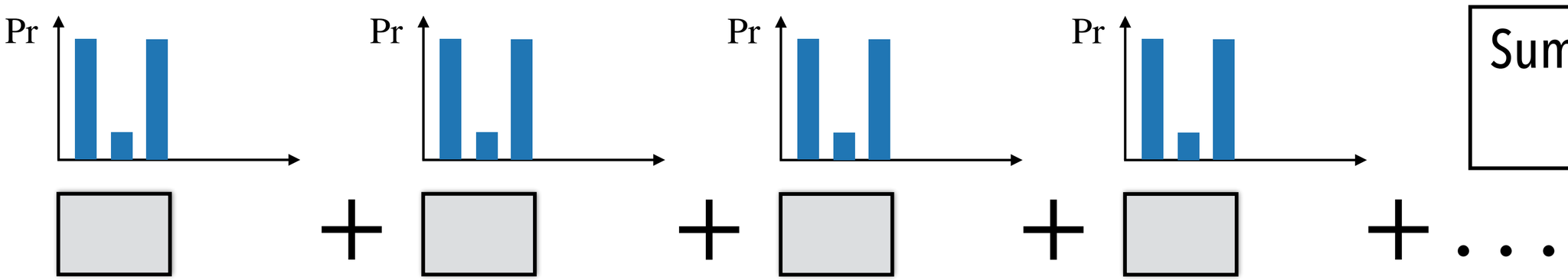
job's execution

=



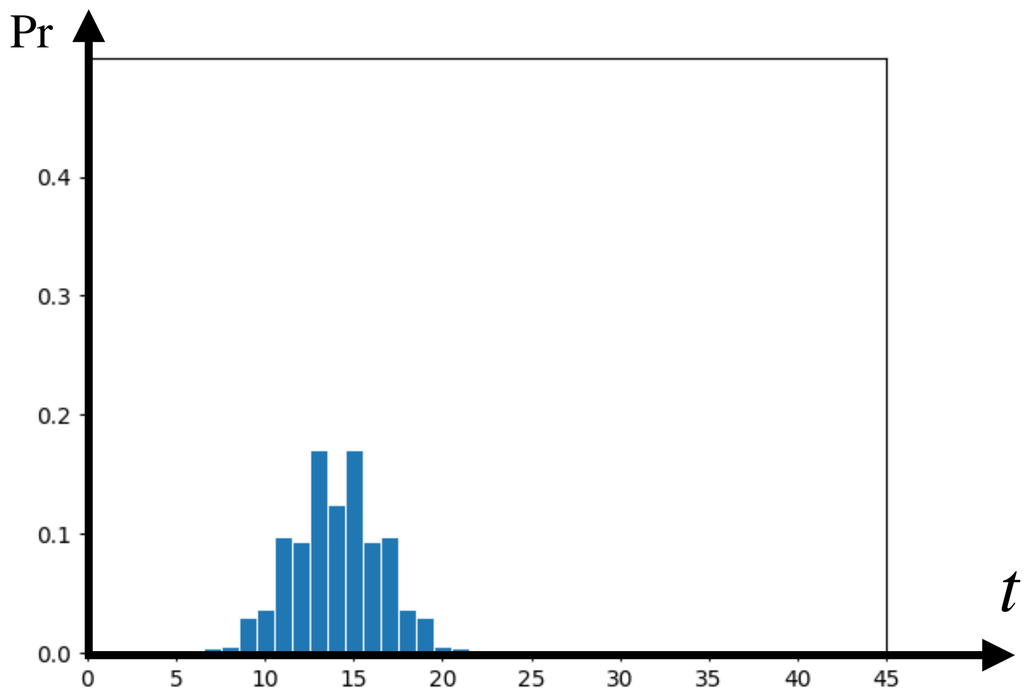
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



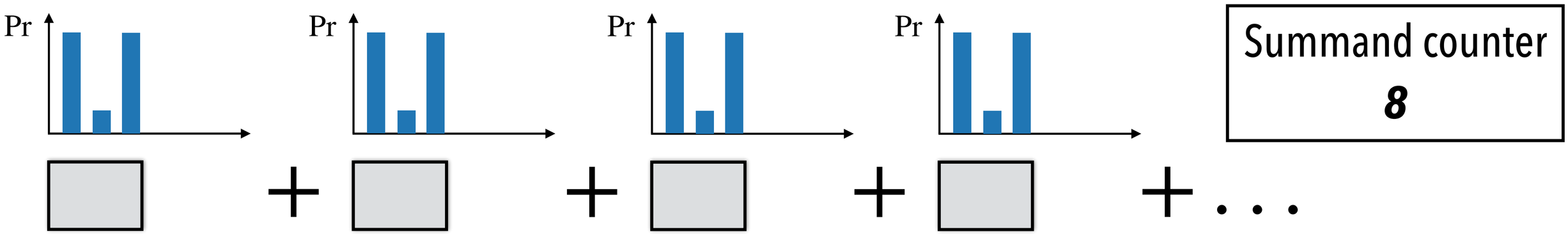
job's execution

=



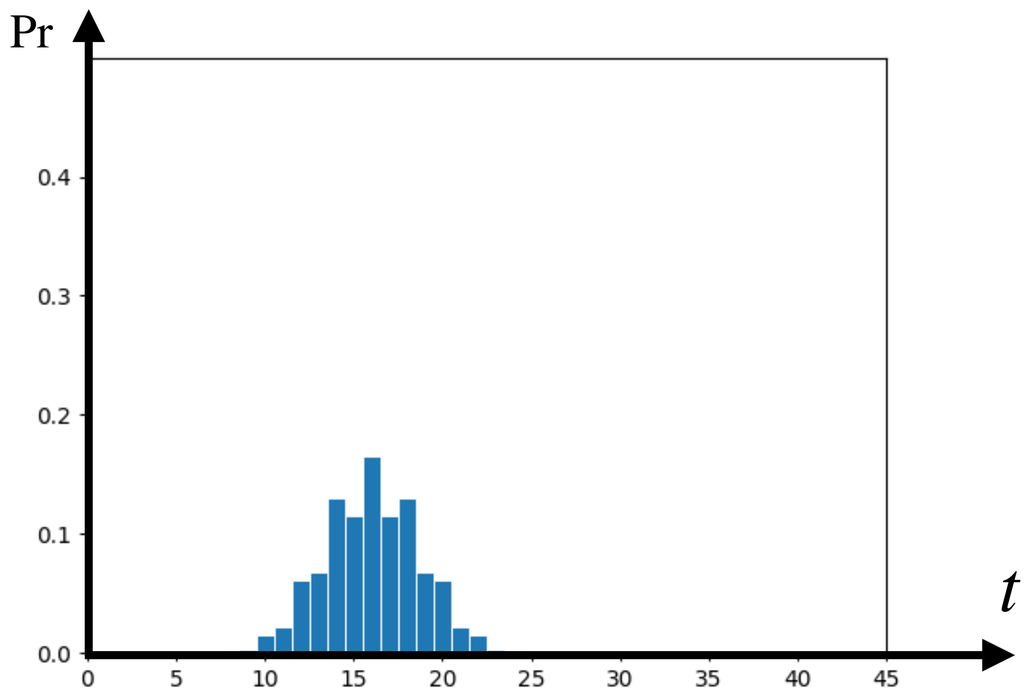
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



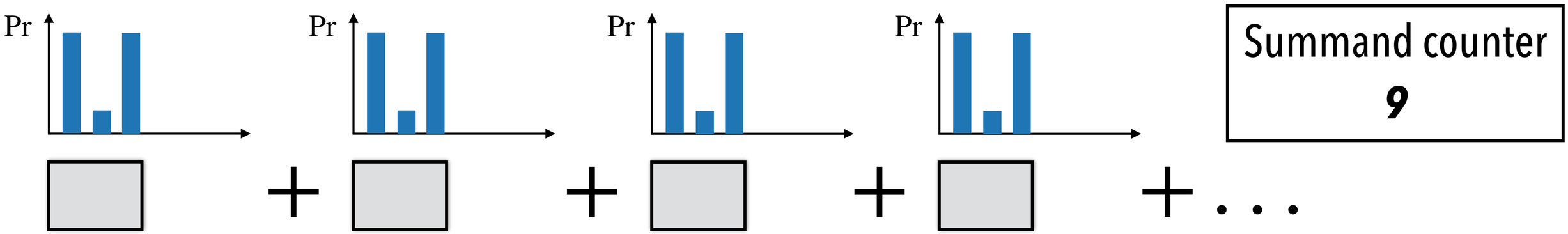
job's execution

=



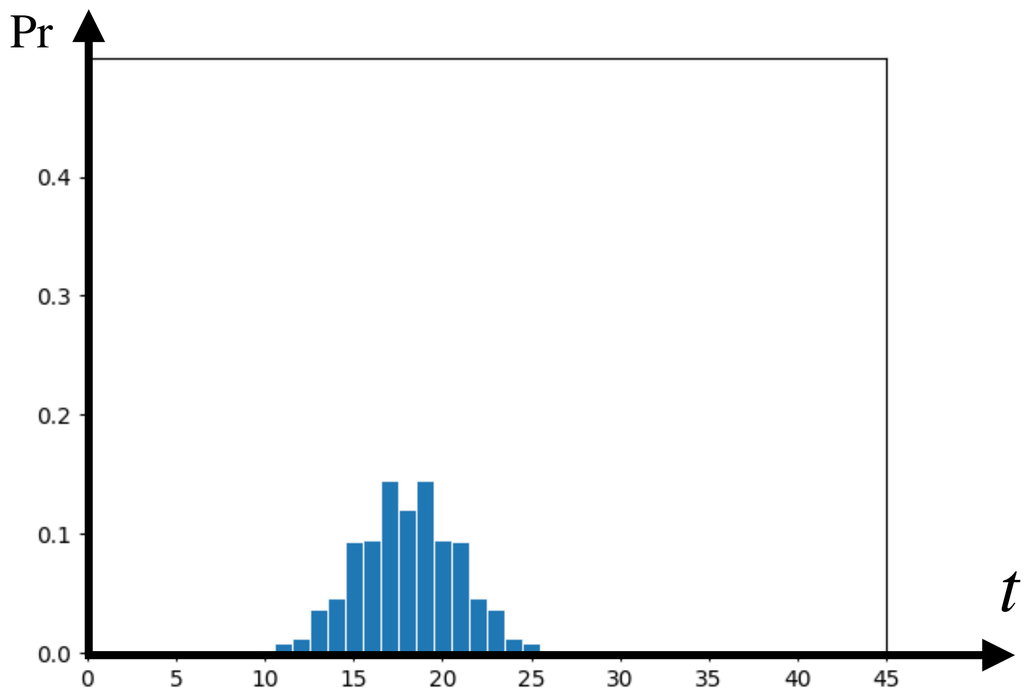
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



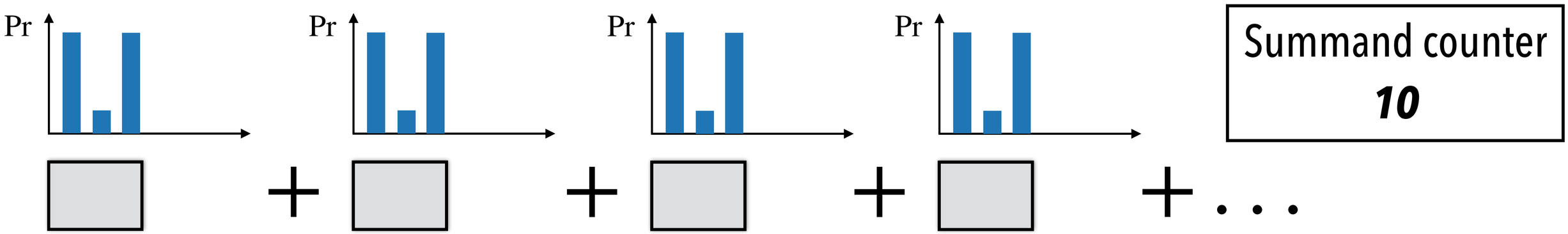
job's execution

=



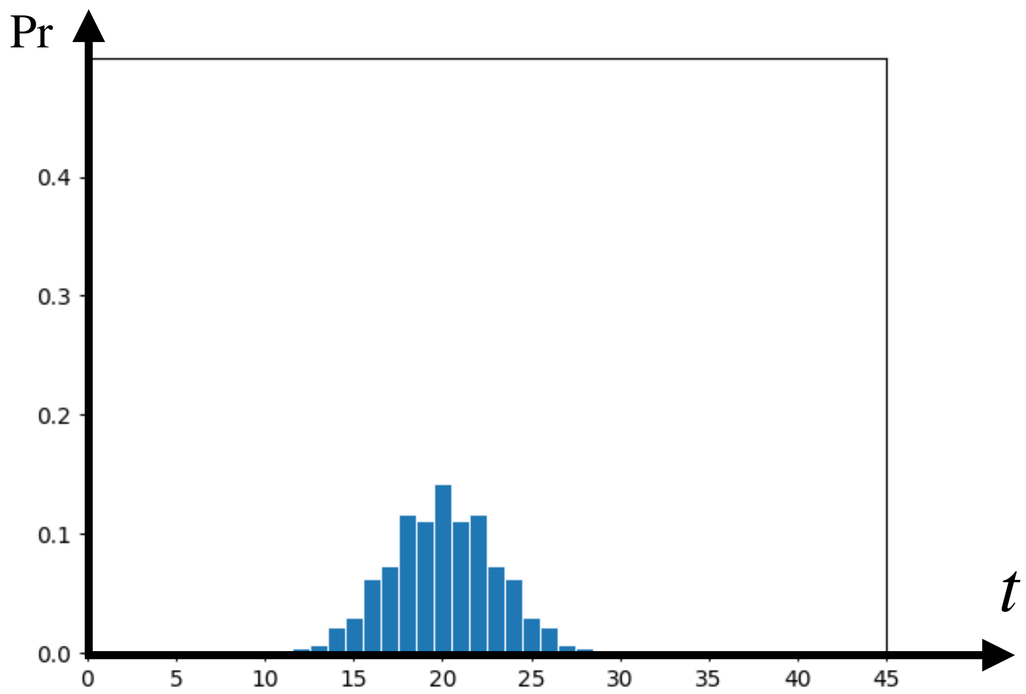
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



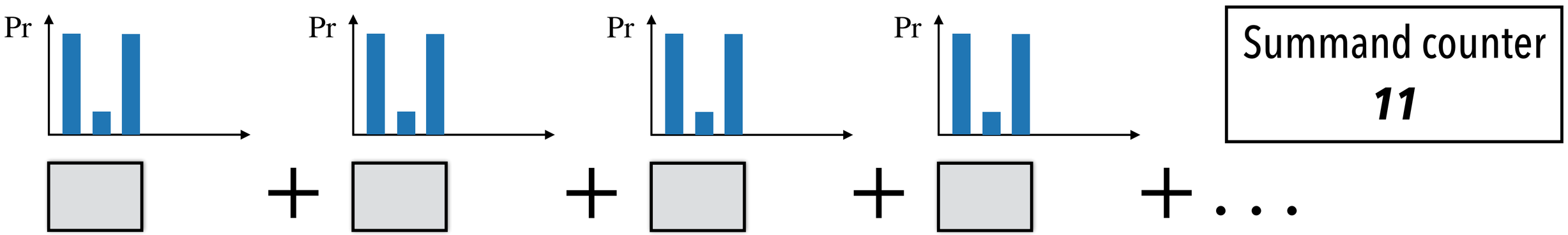
job's execution

=



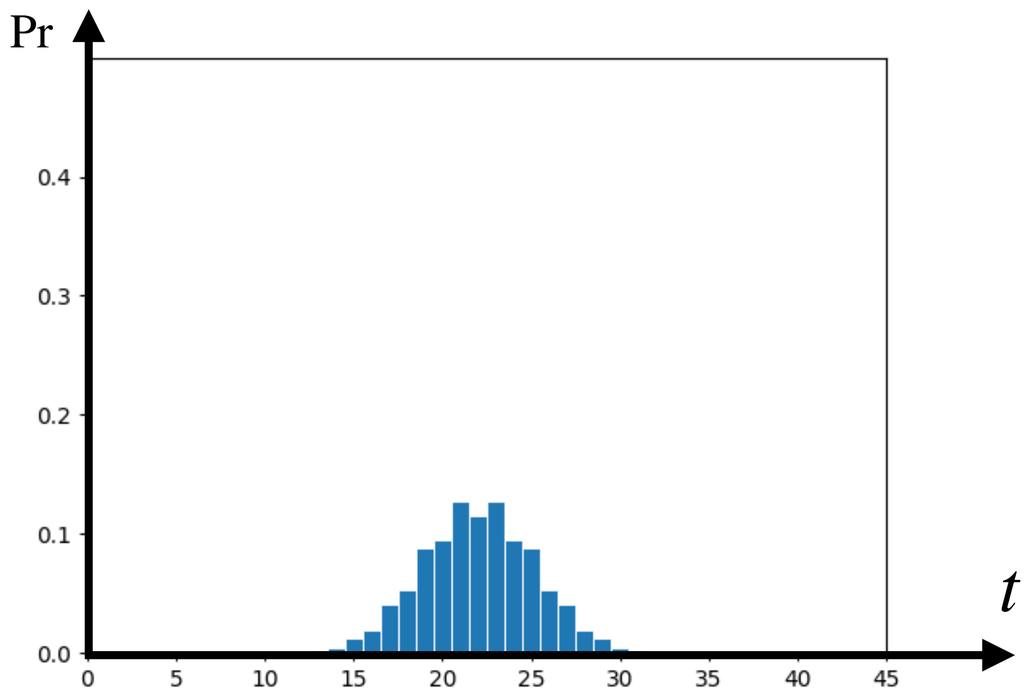
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



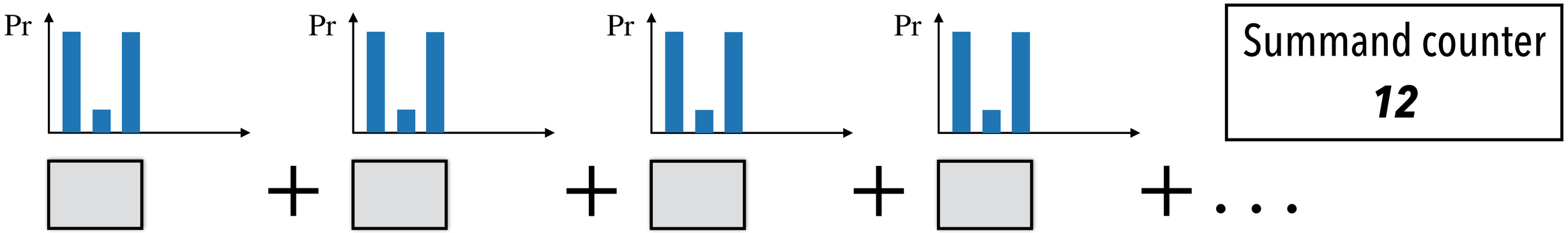
job's execution

=



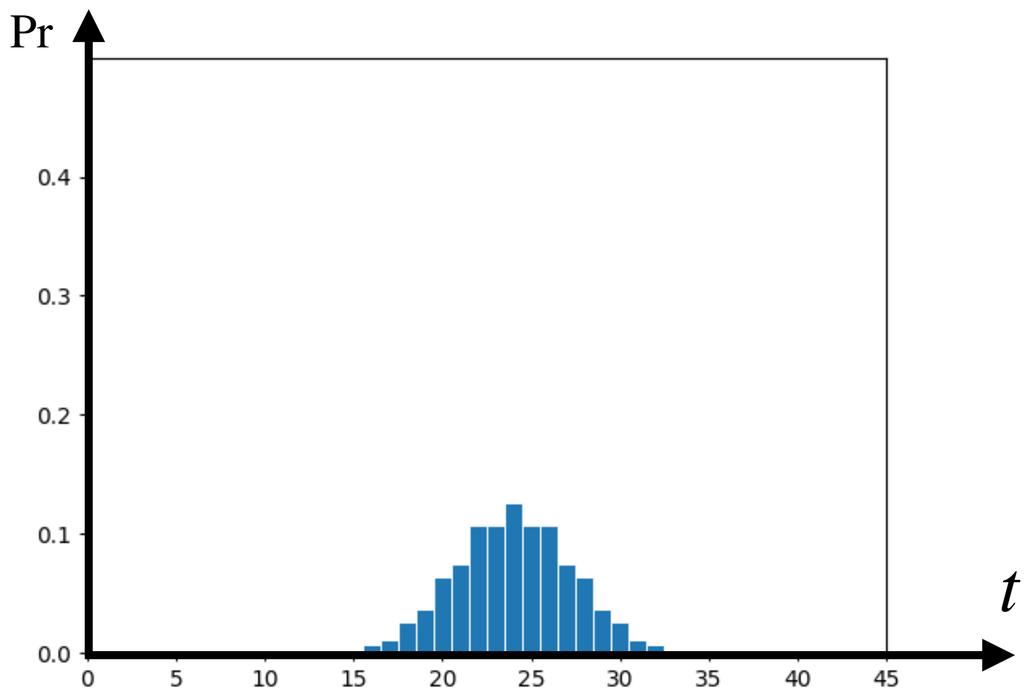
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



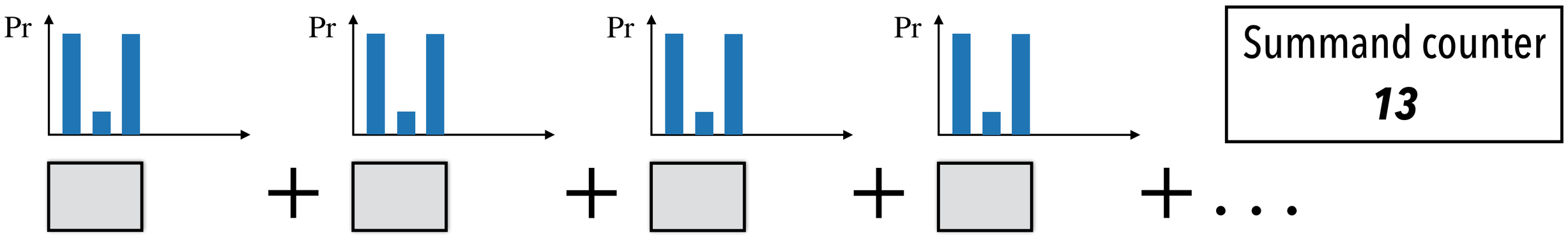
job's execution

=



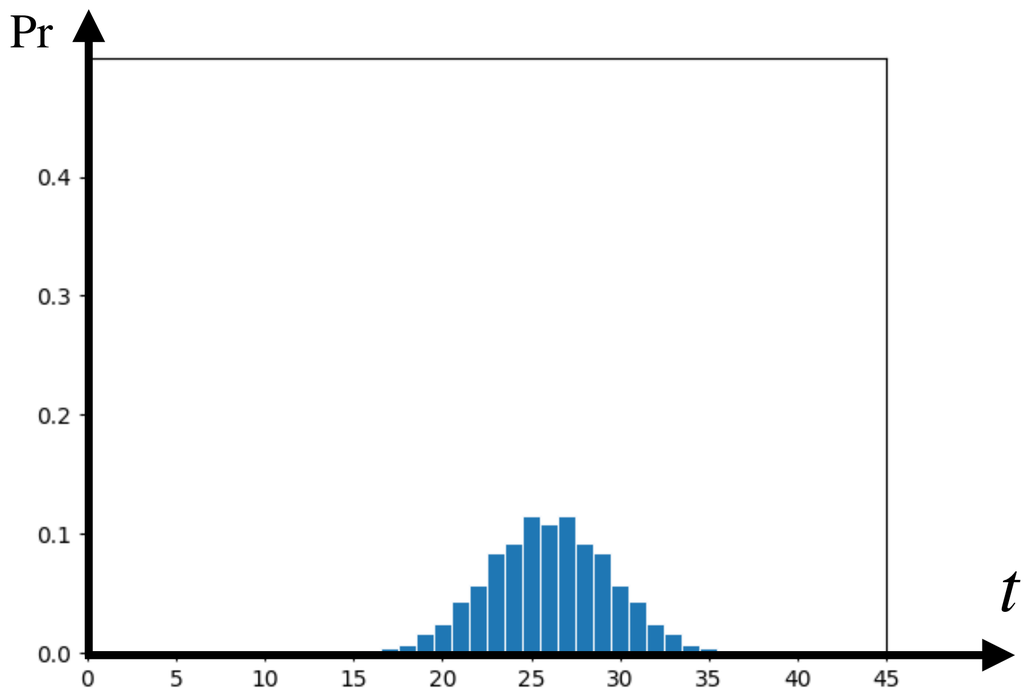
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



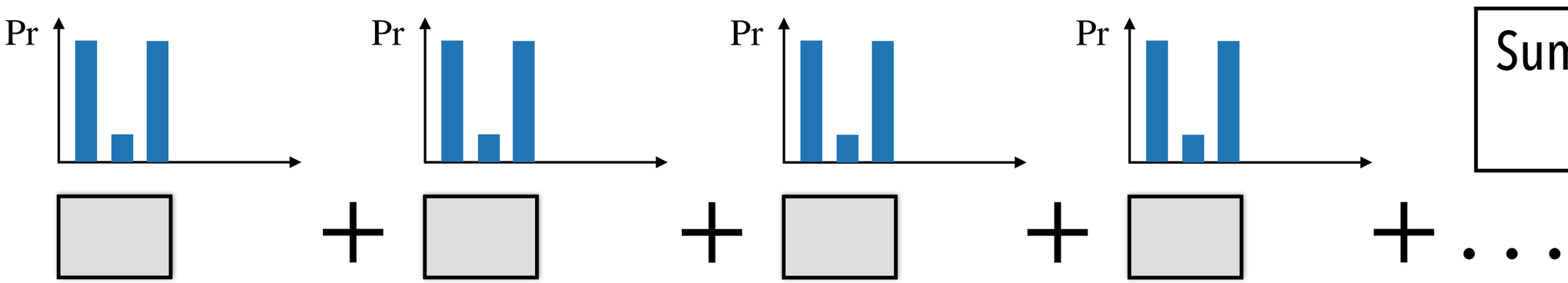
job's execution

=



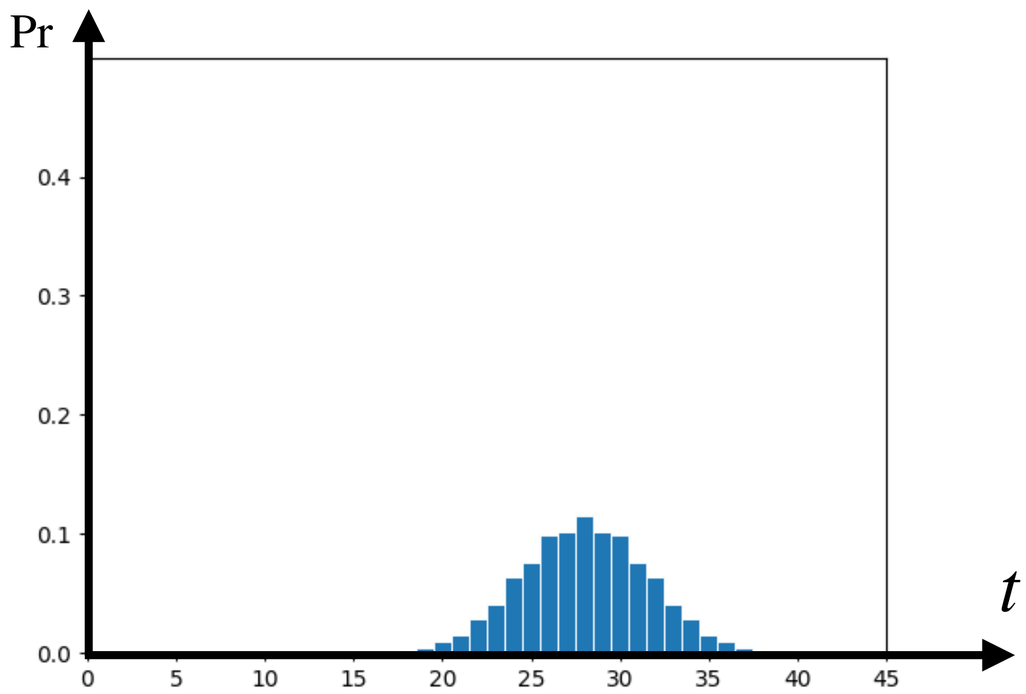
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



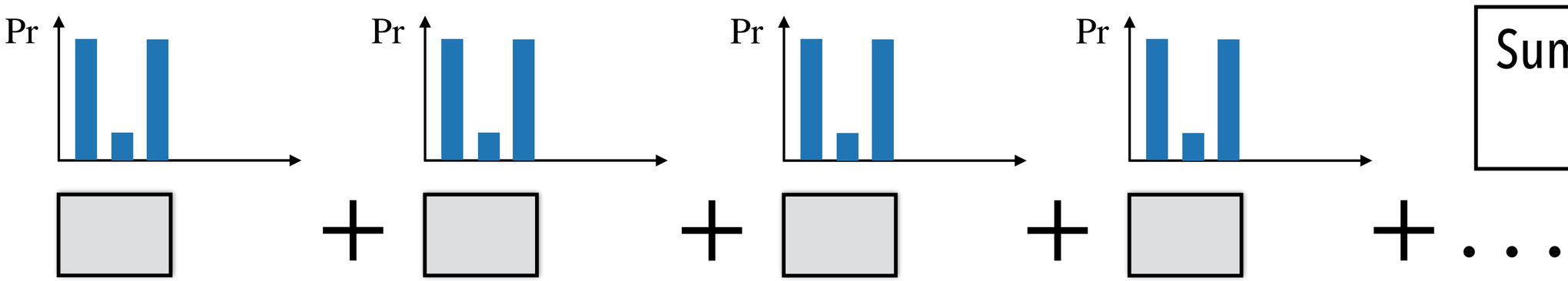
job's execution

=



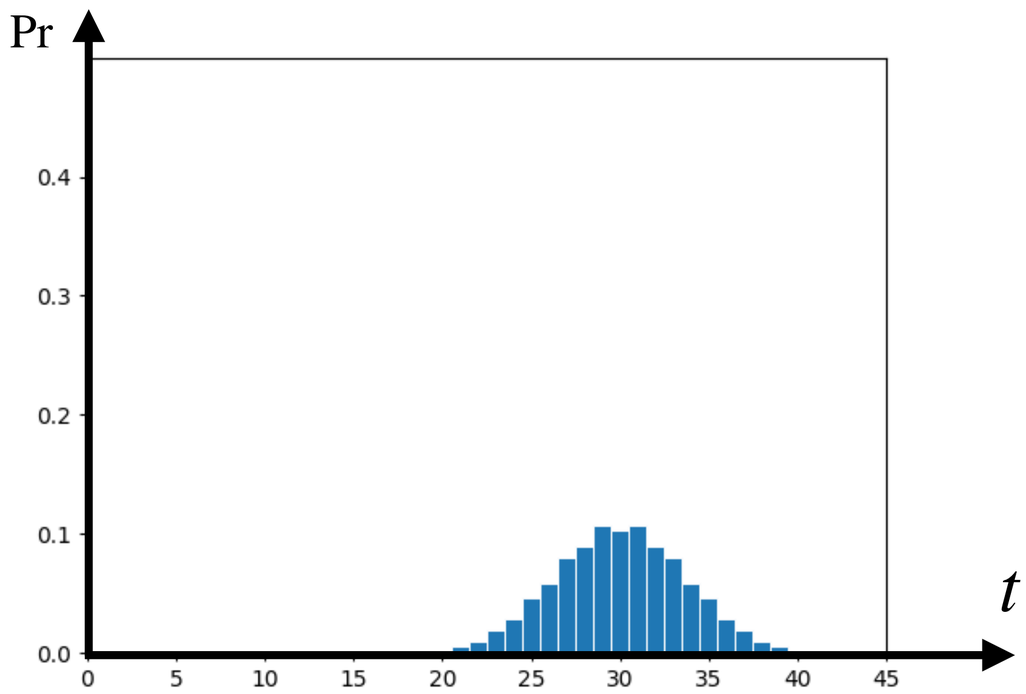
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



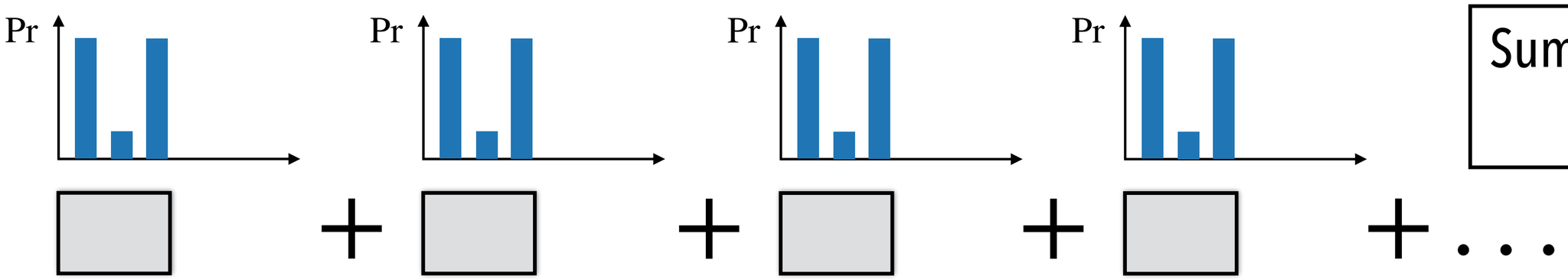
job's execution

=



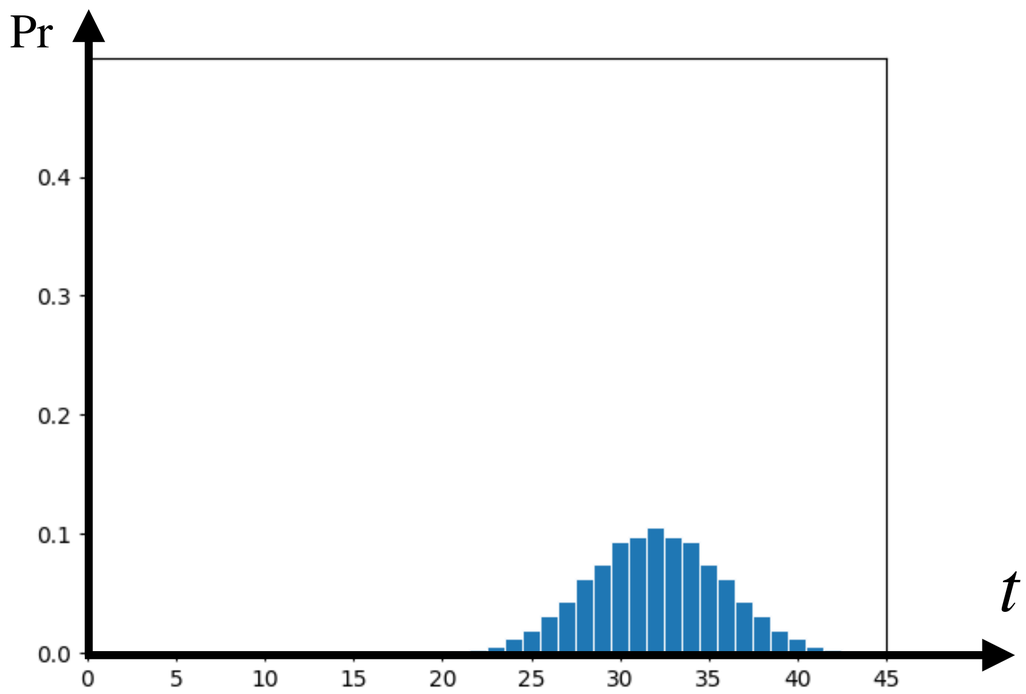
INTUITION BEHIND THE SOLUTION

*Consider summing up the following job distribution **several times**.*



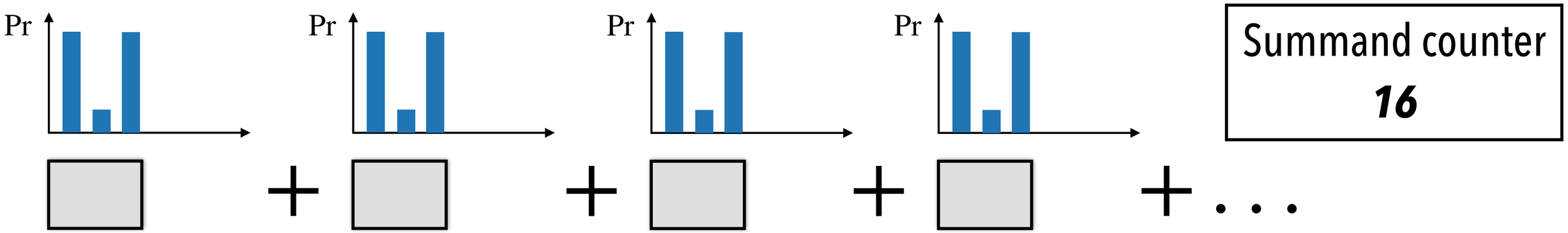
job's execution

=



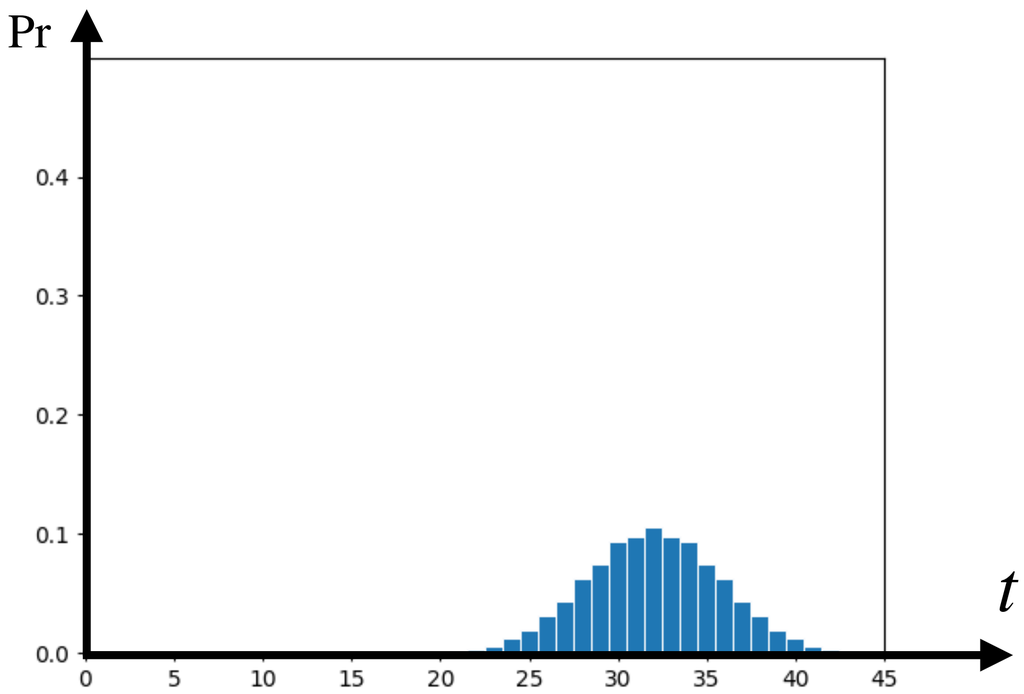
A CONVENIENT PATTERN IS OBSERVED

We conclude that there is something interesting about this summation process.



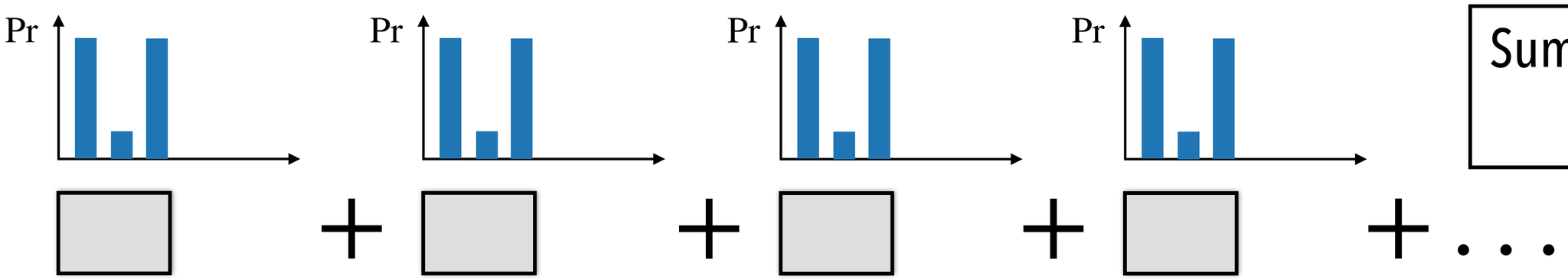
job's execution

=



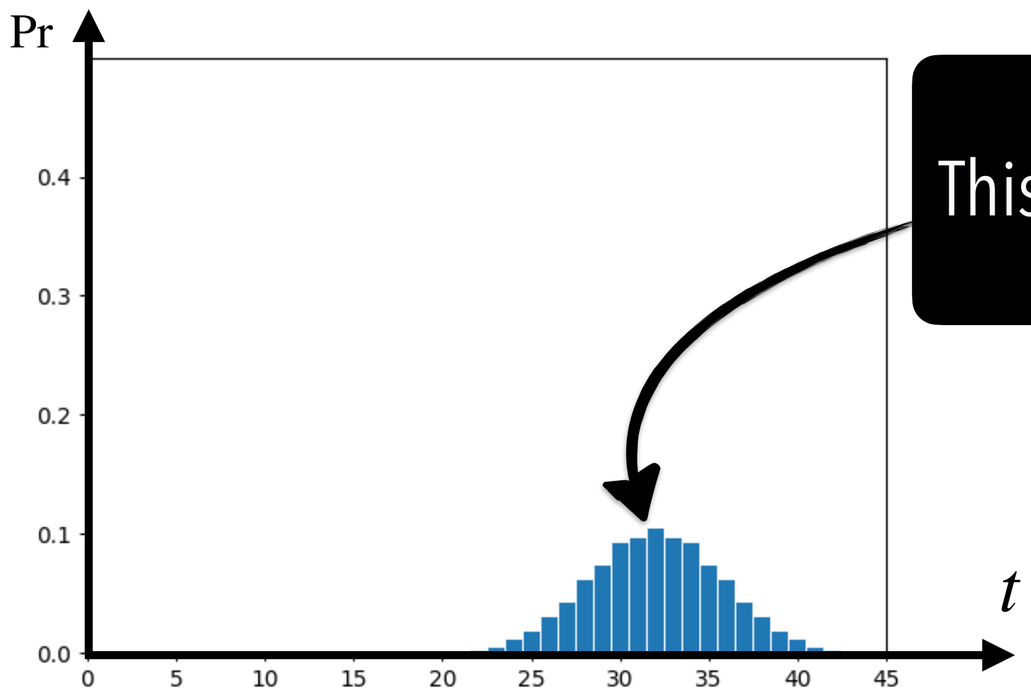
A CONVENIENT PATTERN IS OBSERVED

We conclude that there is something interesting about this summation process.



job's execution

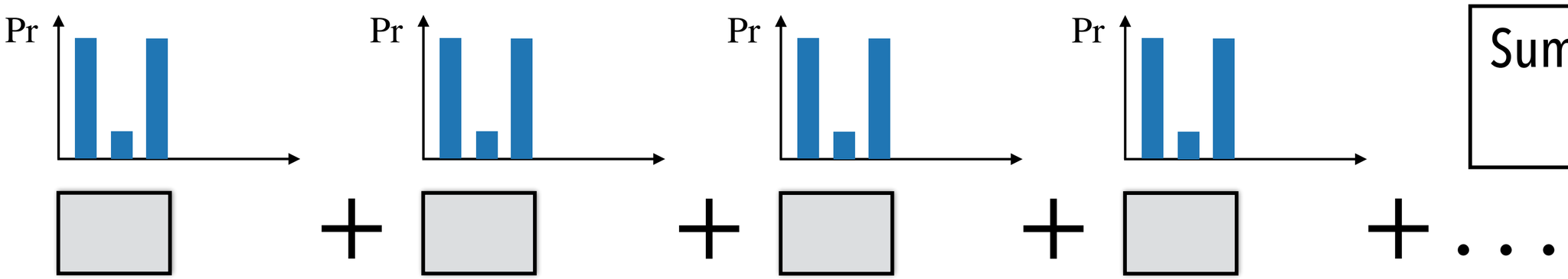
=



This distribution rings a **bell**.

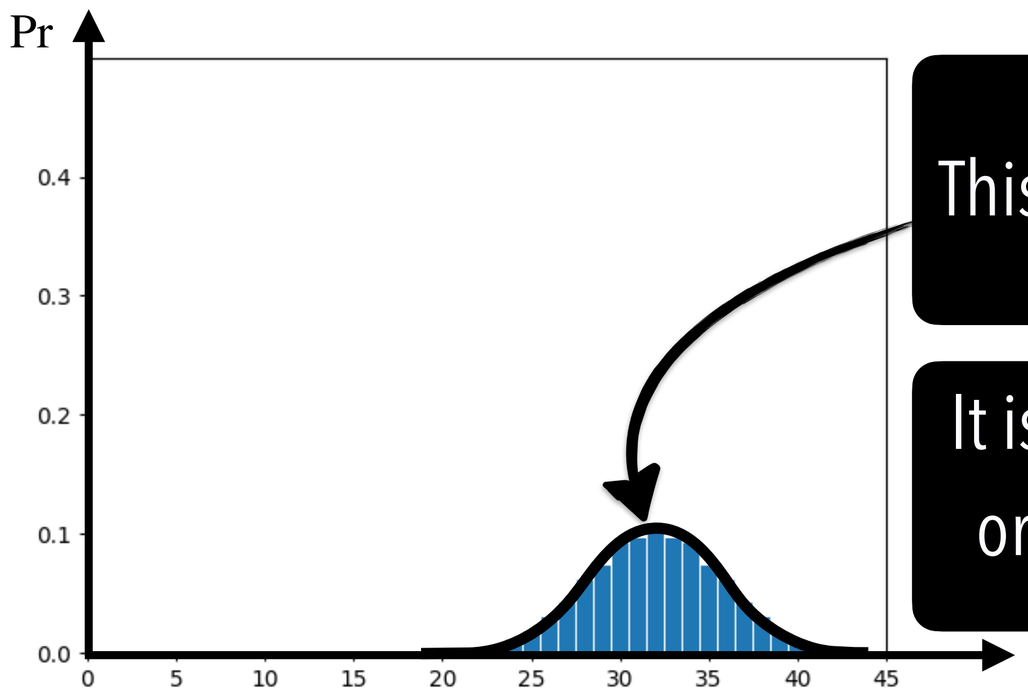
A CONVENIENT PATTERN IS OBSERVED

We conclude that there is something interesting about this summation process.



job's execution

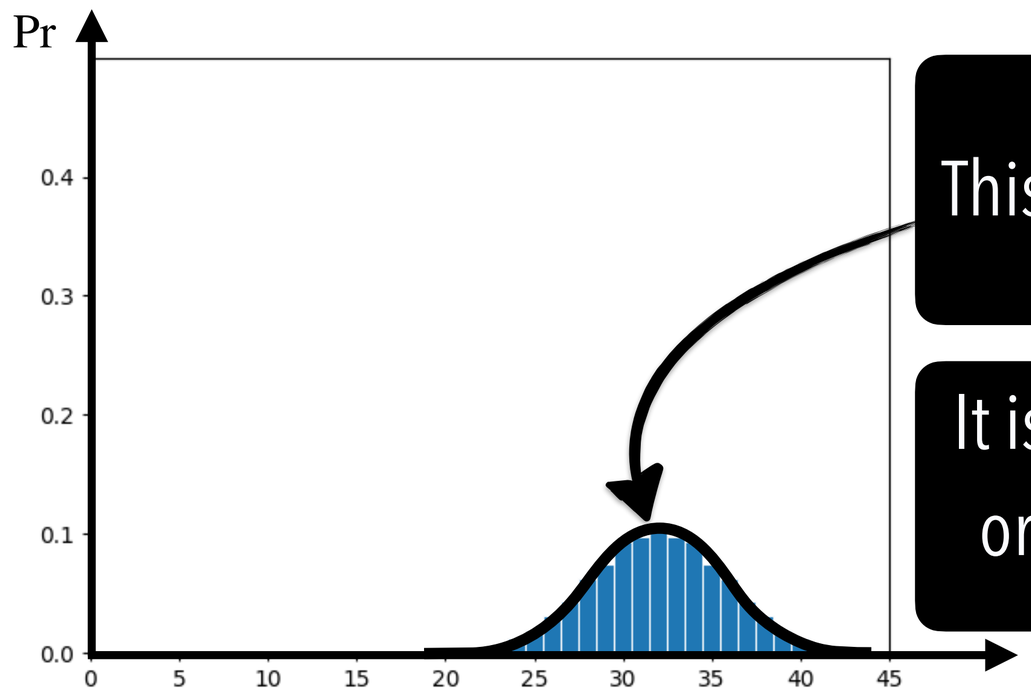
=



This distribution rings a **bell**.

It is known as the **Bell** curve or **Normal distribution.**

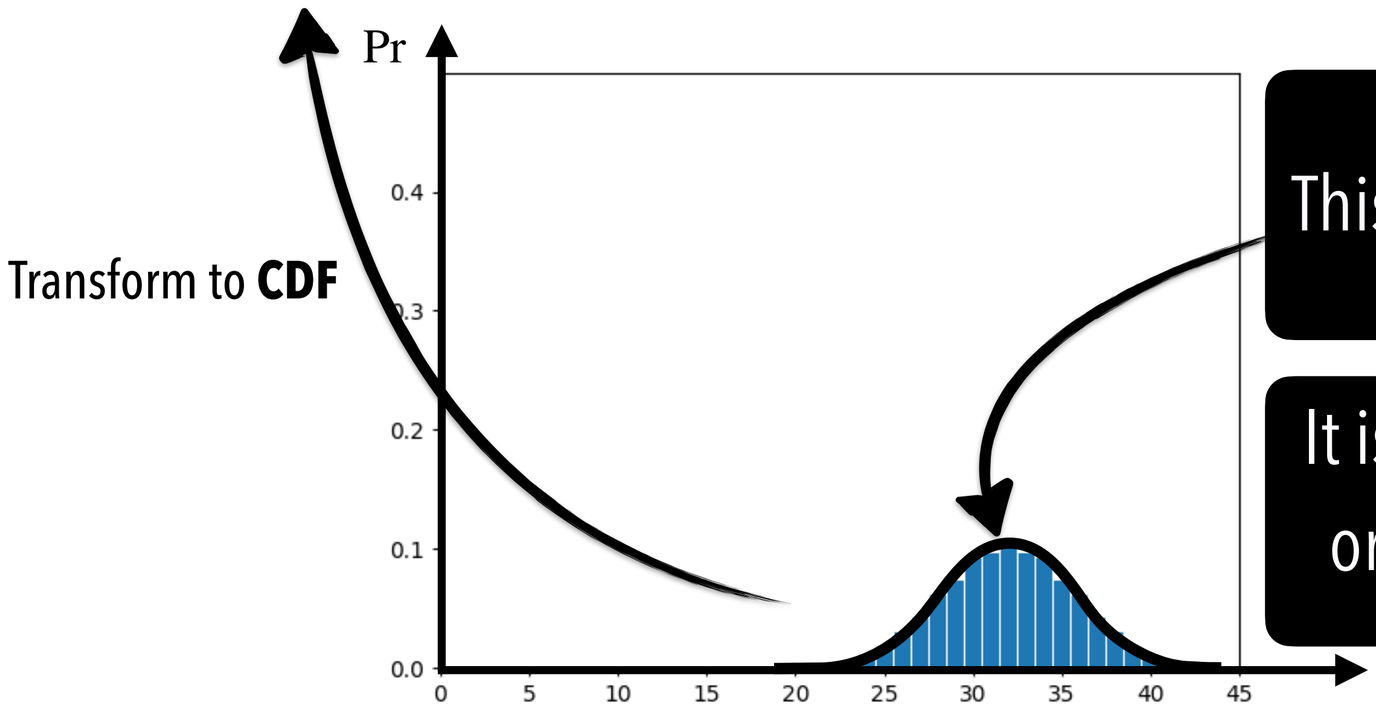
IN MORE DETAIL



This distribution rings a **bell**.

It is known as the **Bell** curve or **Normal distribution.**

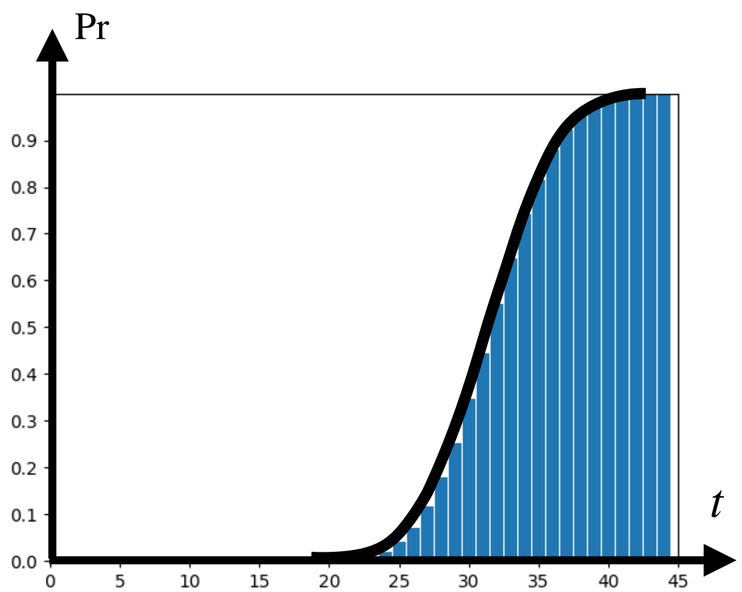
IN MORE DETAIL



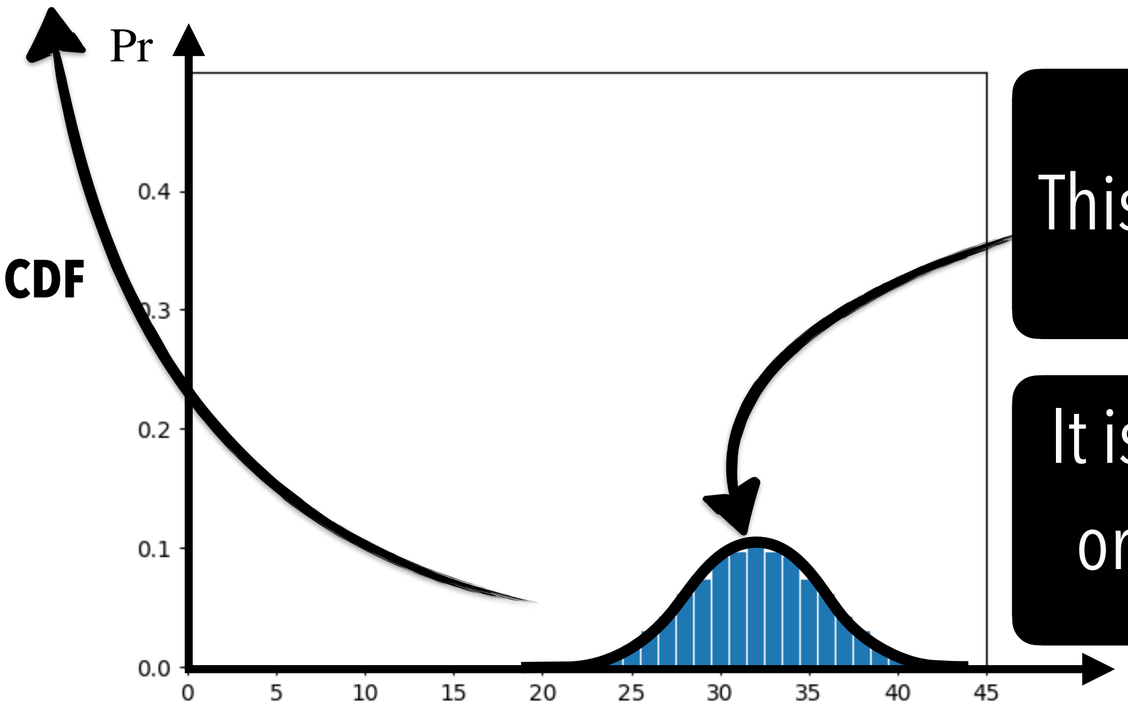
This distribution rings a **bell**.

It is known as the **Bell** curve or **Normal distribution.**

IN MORE DETAIL



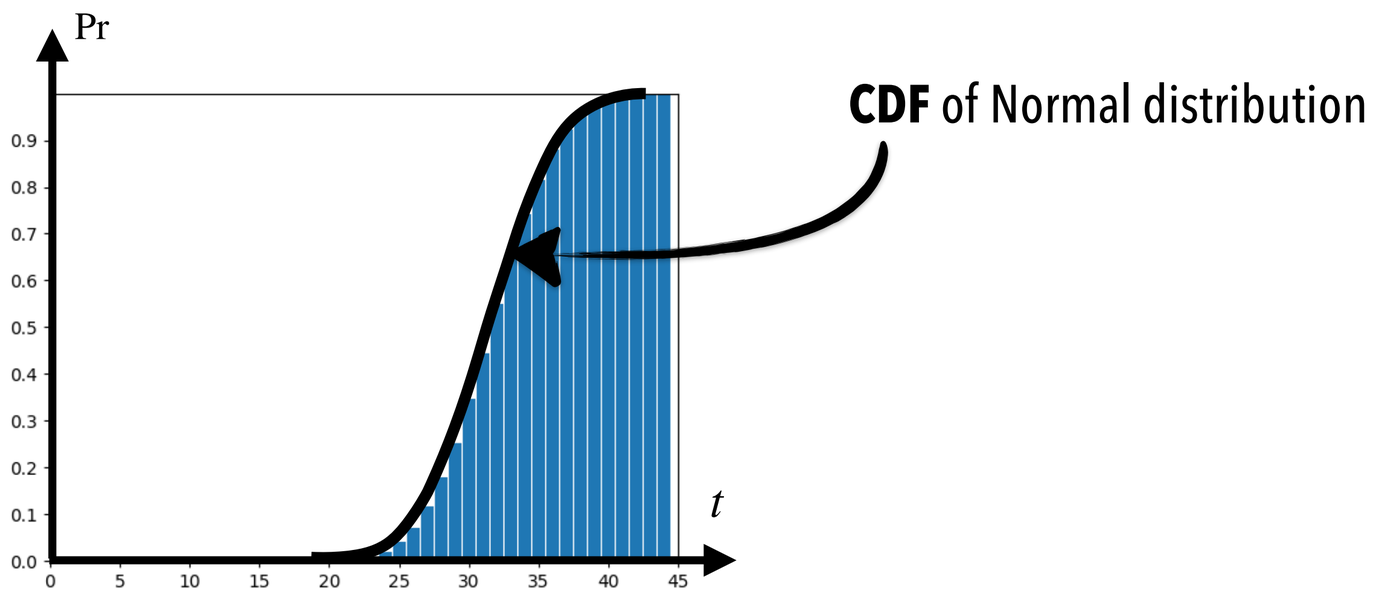
Transform to **CDF**



This distribution rings a **bell**.

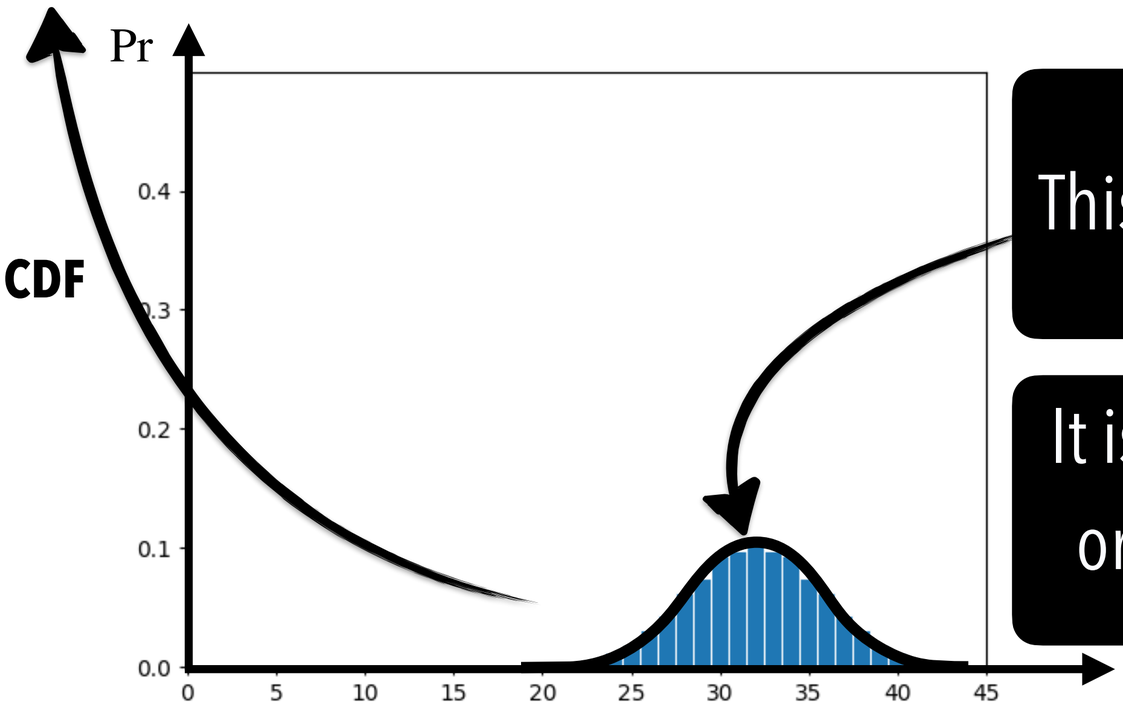
It is known as the **Bell** curve or **Normal distribution.**

IN MORE DETAIL



CDF of Normal distribution

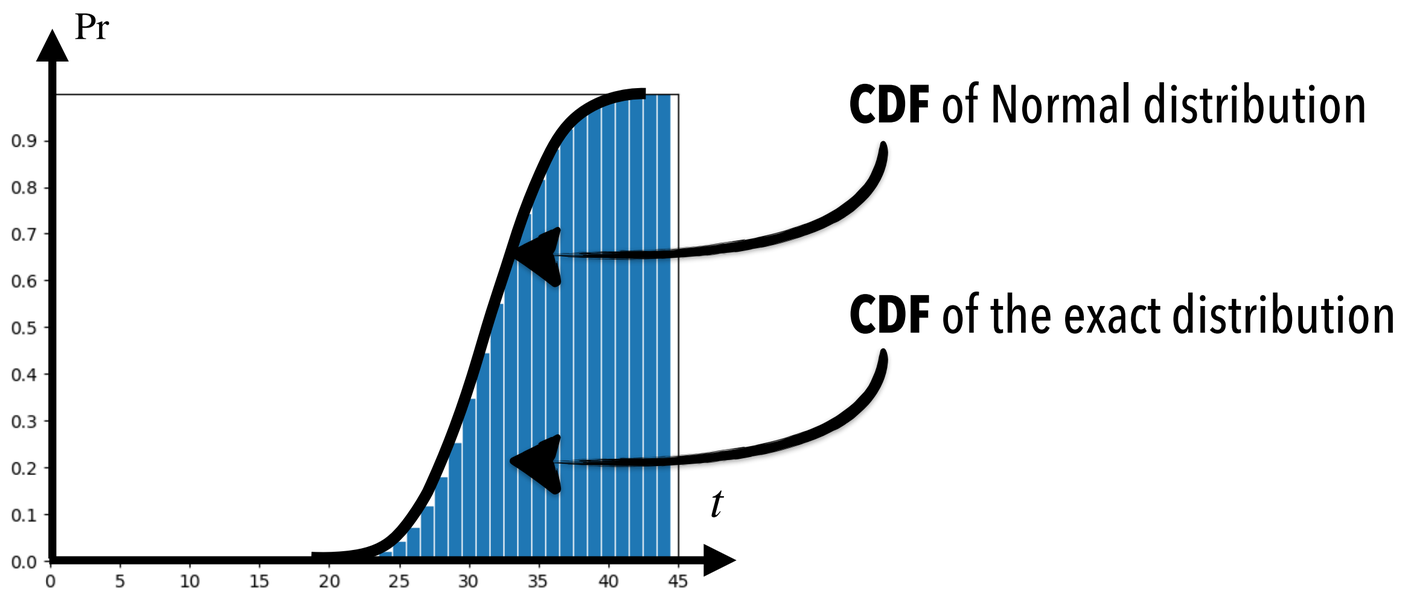
Transform to **CDF**



This distribution rings a **bell**.

It is known as the **Bell** curve or **Normal distribution.**

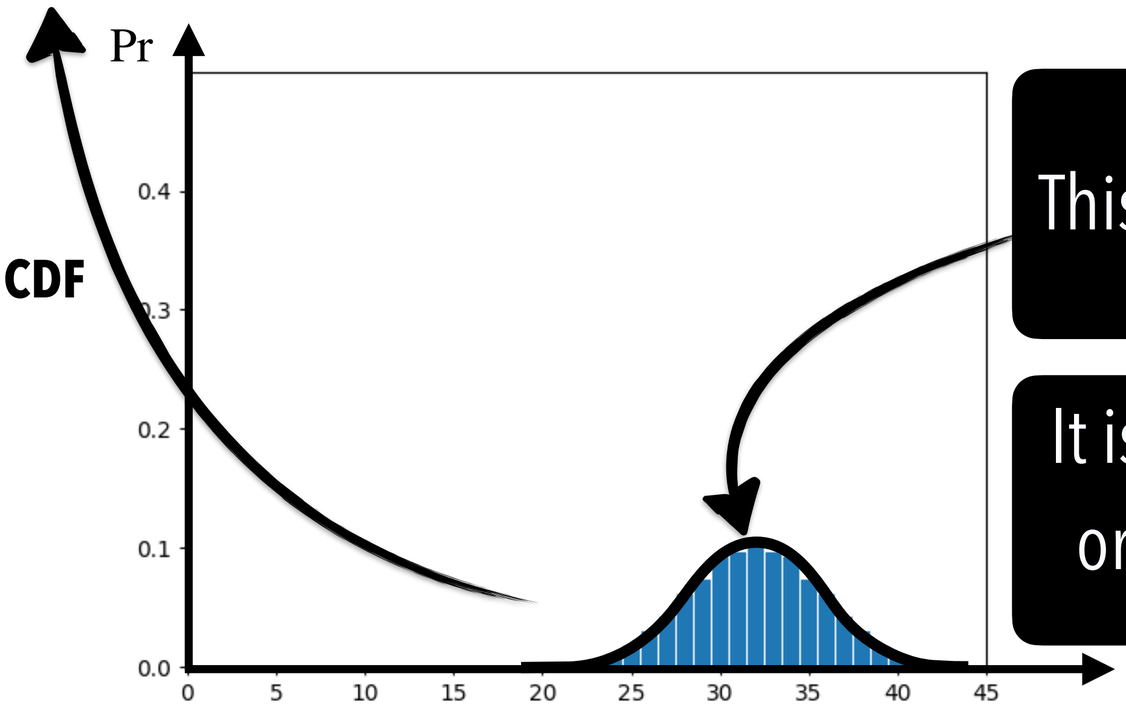
IN MORE DETAIL



CDF of Normal distribution

CDF of the exact distribution

Transform to **CDF**

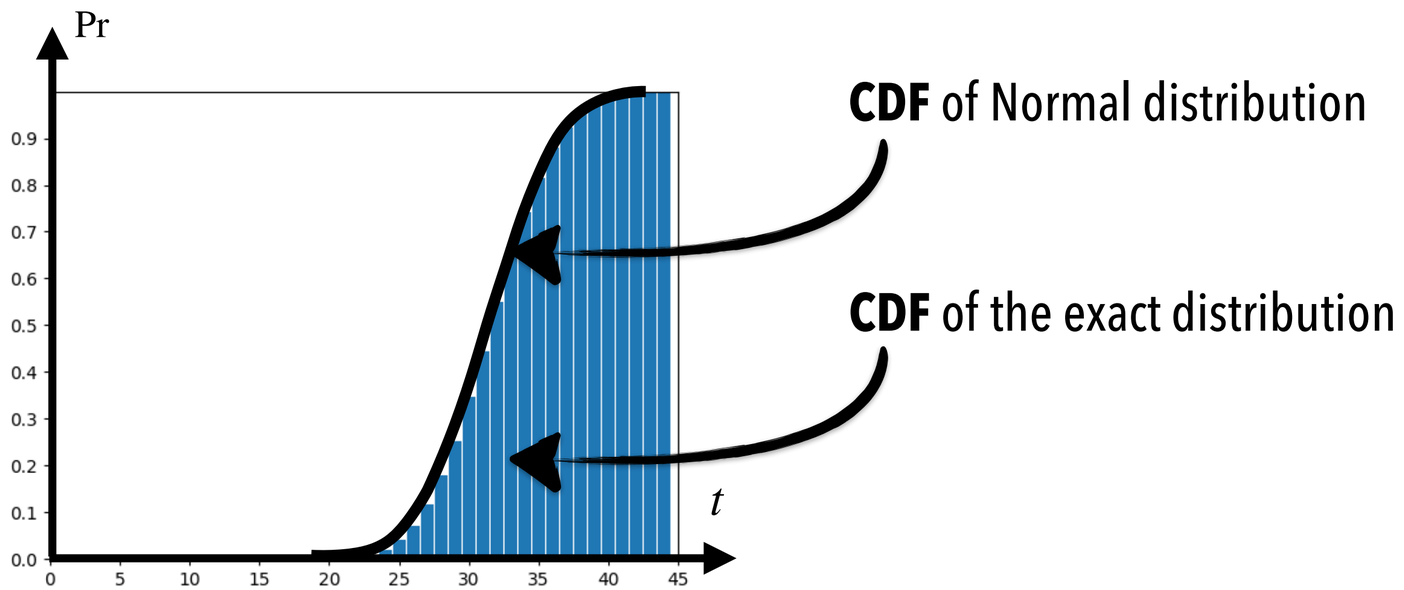


This distribution rings a **bell**.

It is known as the **Bell** curve or **Normal distribution.**

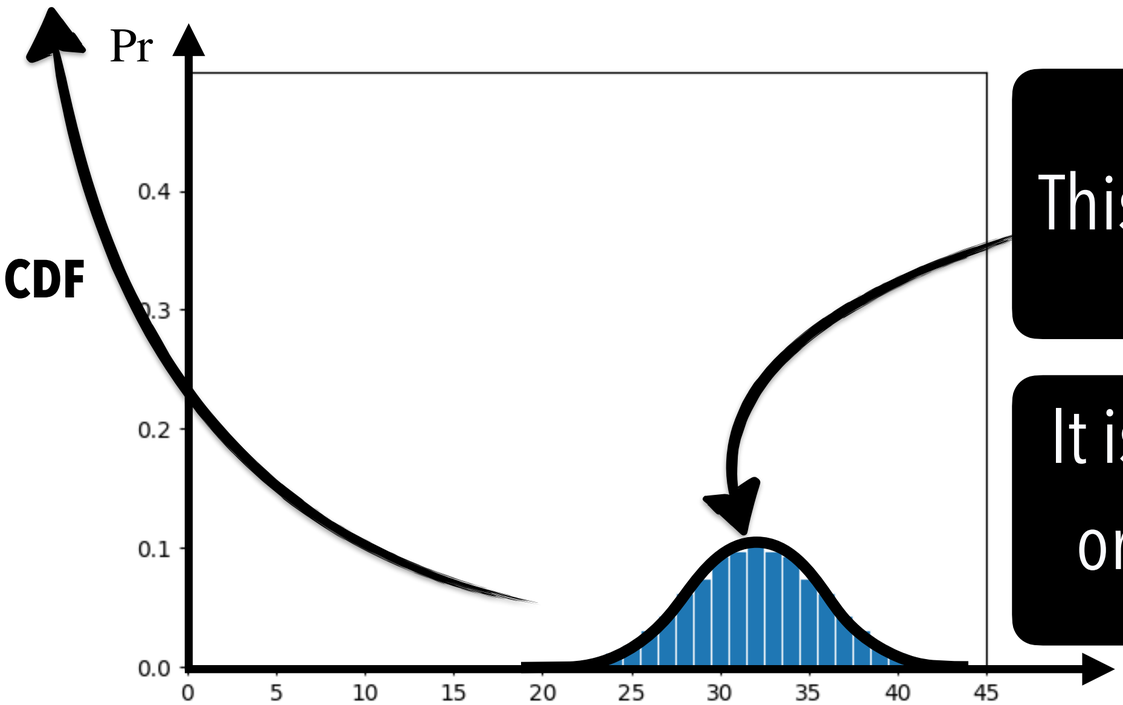
IN MORE DETAIL

*The **Cumulative Distribution Function** (CDF) of the sum of discrete independent random variables **converges** to the CDF of **Normal distribution*** (known as the Lyapunov Central Limit Theorem).



CDF of Normal distribution

CDF of the exact distribution



Transform to **CDF**

This distribution rings a **bell**.

It is known as the **Bell** curve or **Normal distribution.**

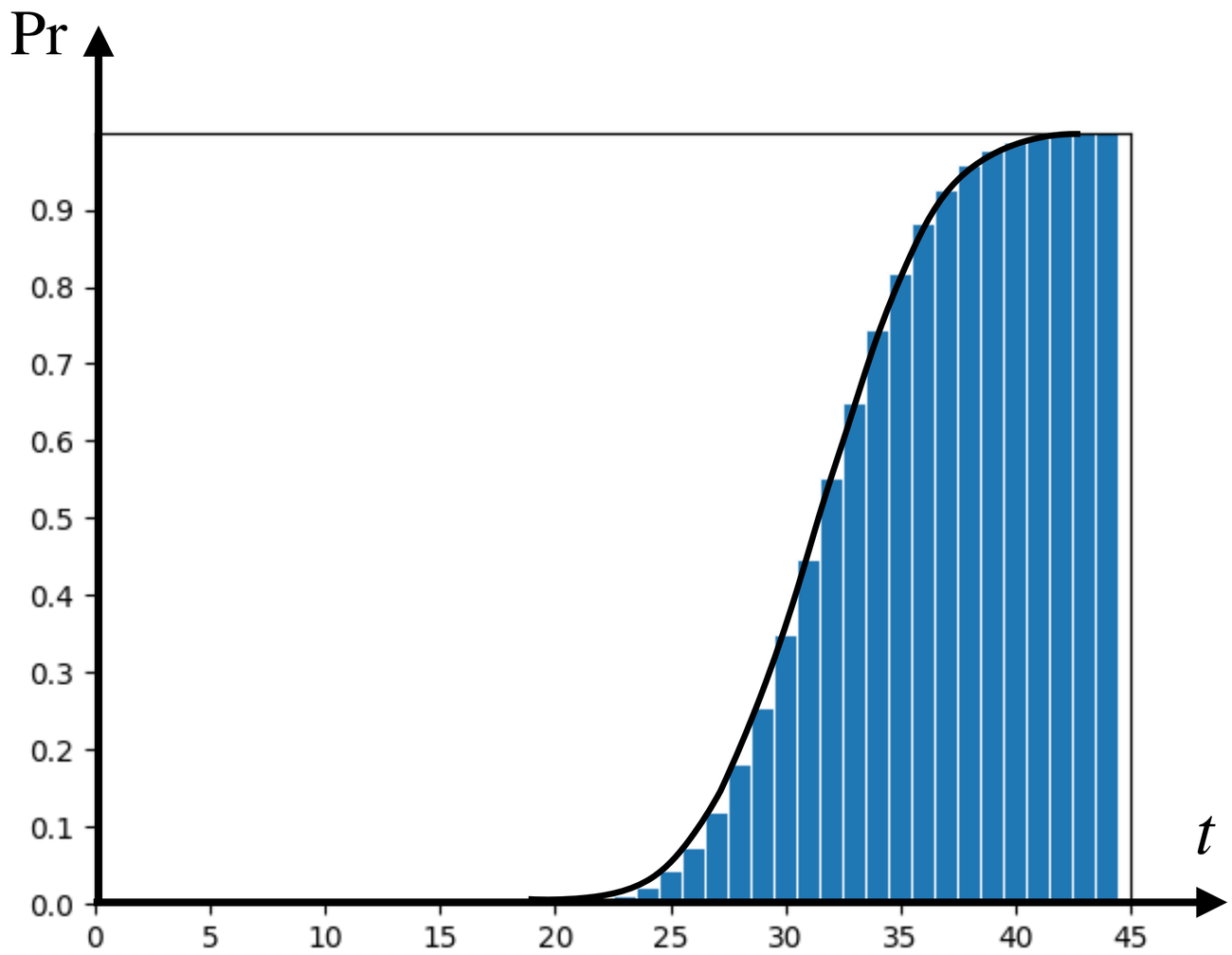
IS THE CDF OF NORMAL DISTRIBUTION SAFE?

IS THE CDF OF NORMAL DISTRIBUTION SAFE?

*The CDF of the normal distribution is **not safe** for analysis of real-time systems.*

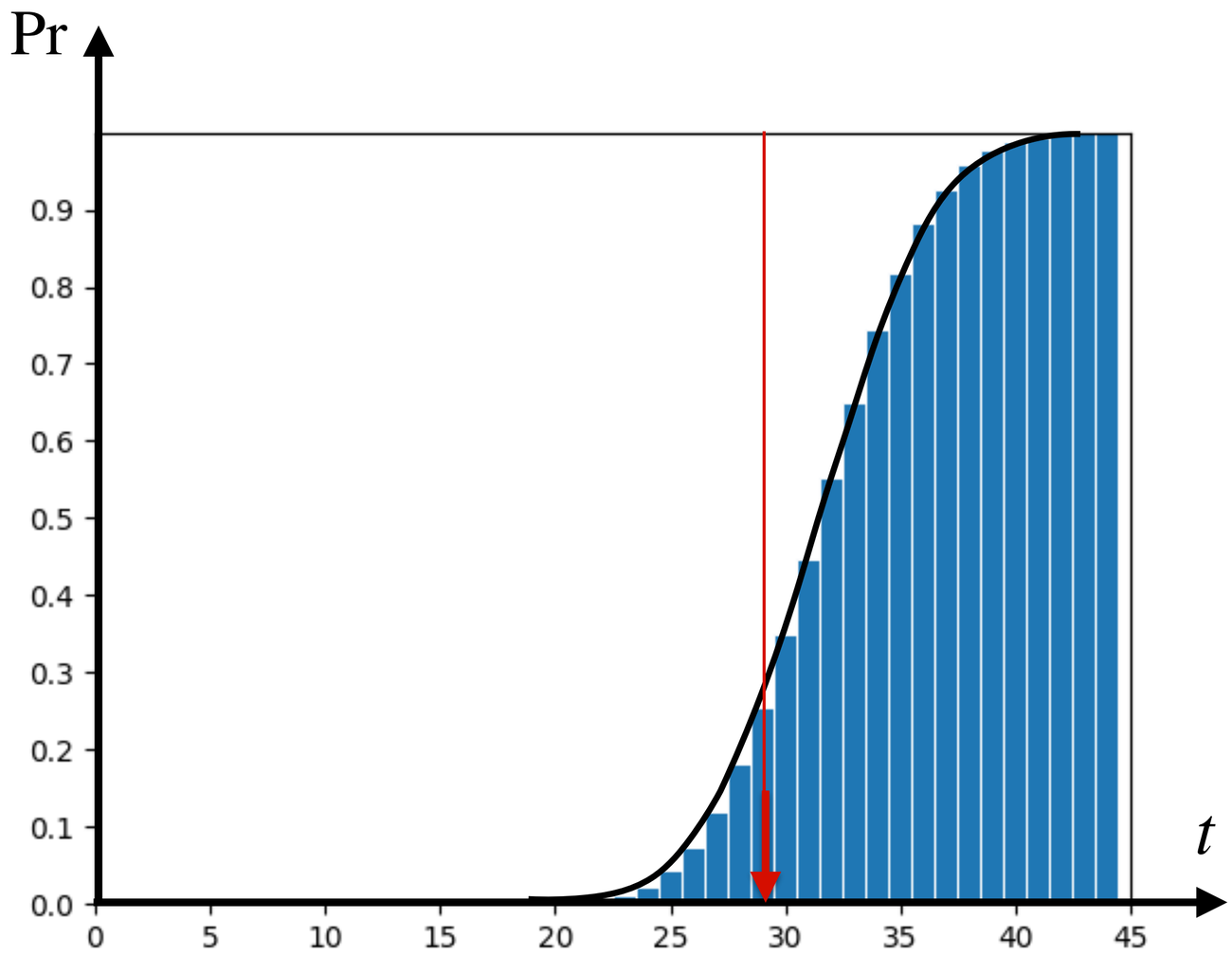
IS THE CDF OF NORMAL DISTRIBUTION SAFE?

*The CDF of the normal distribution is **not safe** for analysis of real-time systems.*



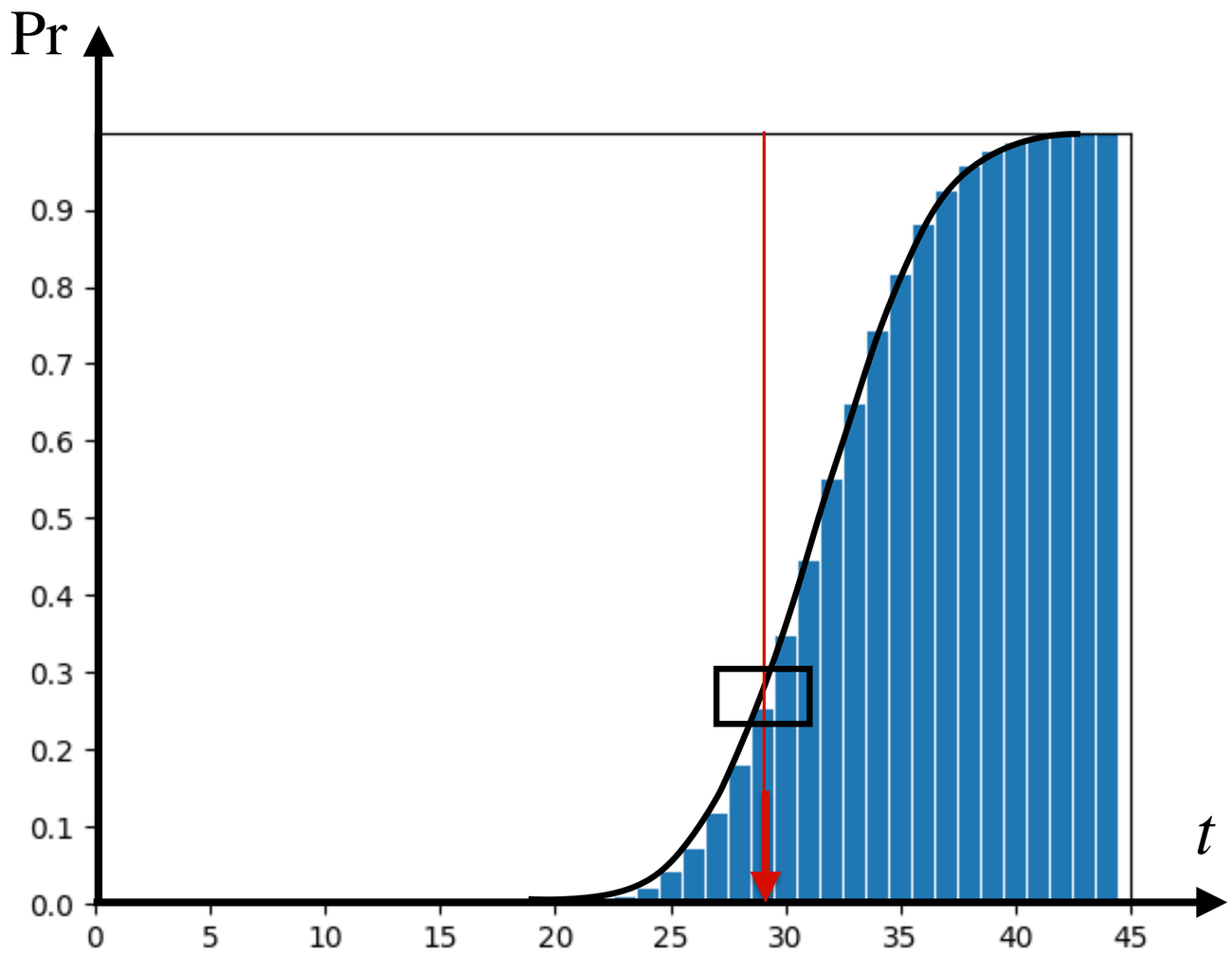
IS THE CDF OF NORMAL DISTRIBUTION SAFE?

*The CDF of the normal distribution is **not safe** for analysis of real-time systems.*



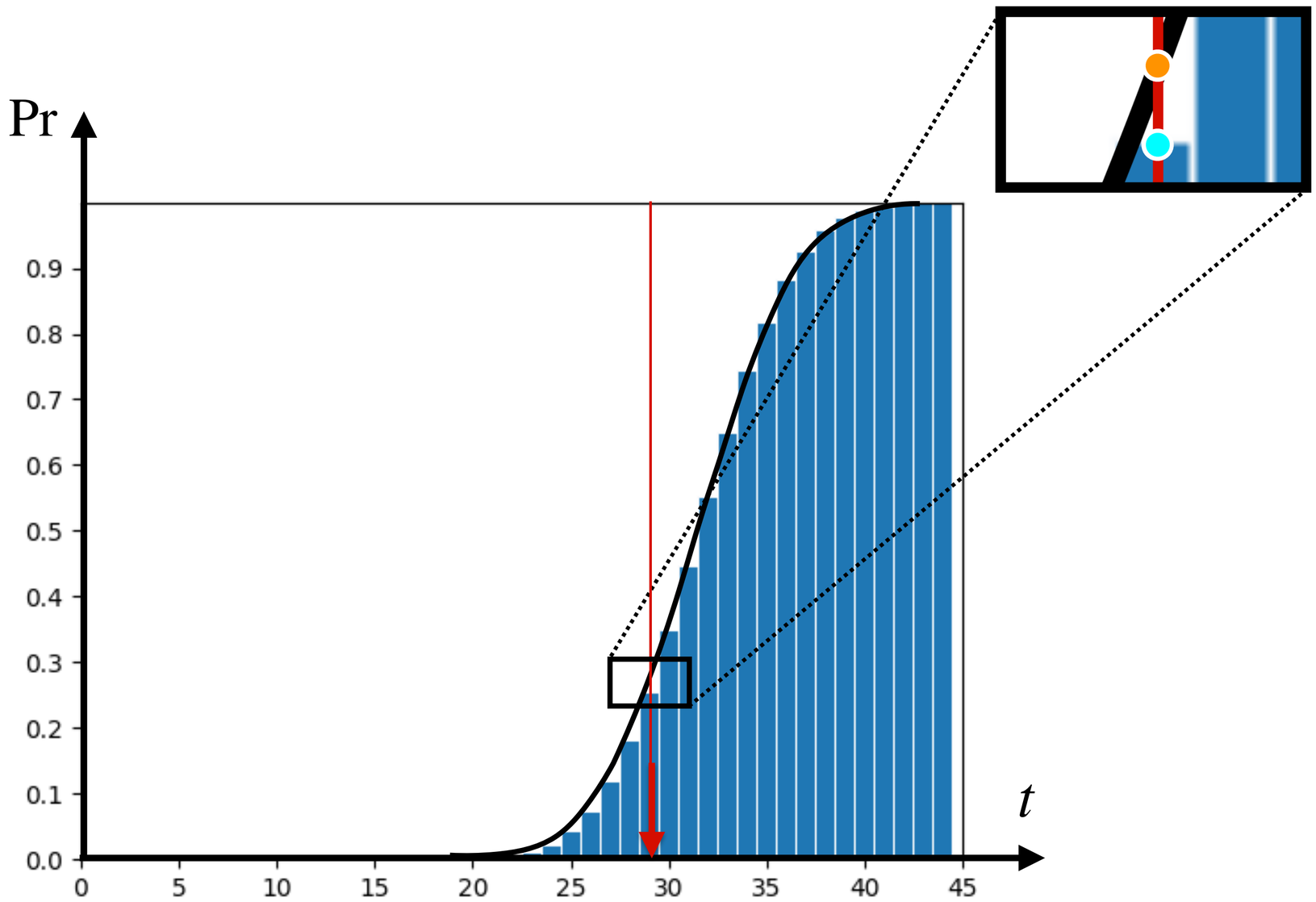
IS THE CDF OF NORMAL DISTRIBUTION SAFE?

*The CDF of the normal distribution is **not safe** for analysis of real-time systems.*



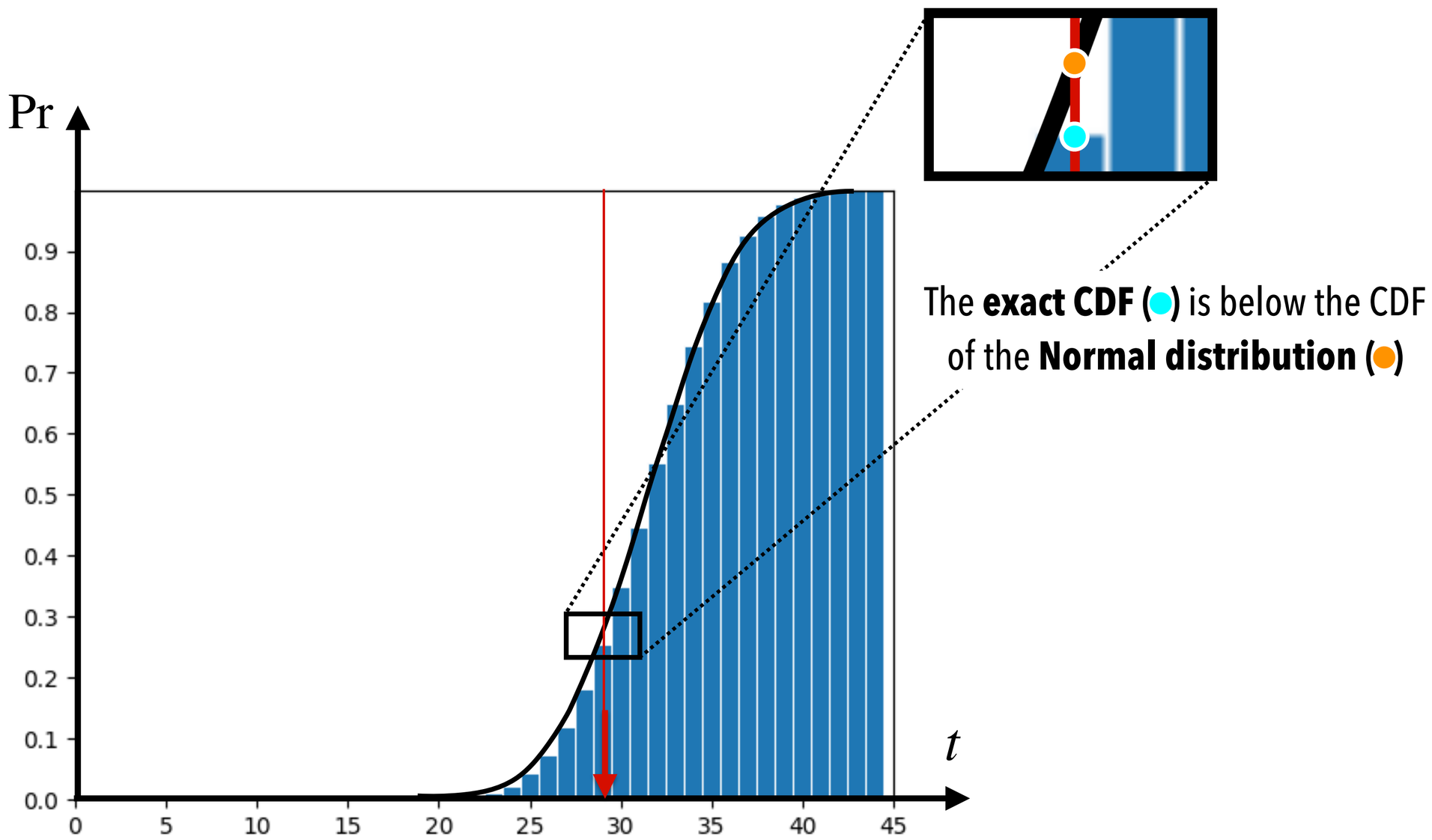
IS THE CDF OF NORMAL DISTRIBUTION SAFE?

*The CDF of the normal distribution is **not safe** for analysis of real-time systems.*



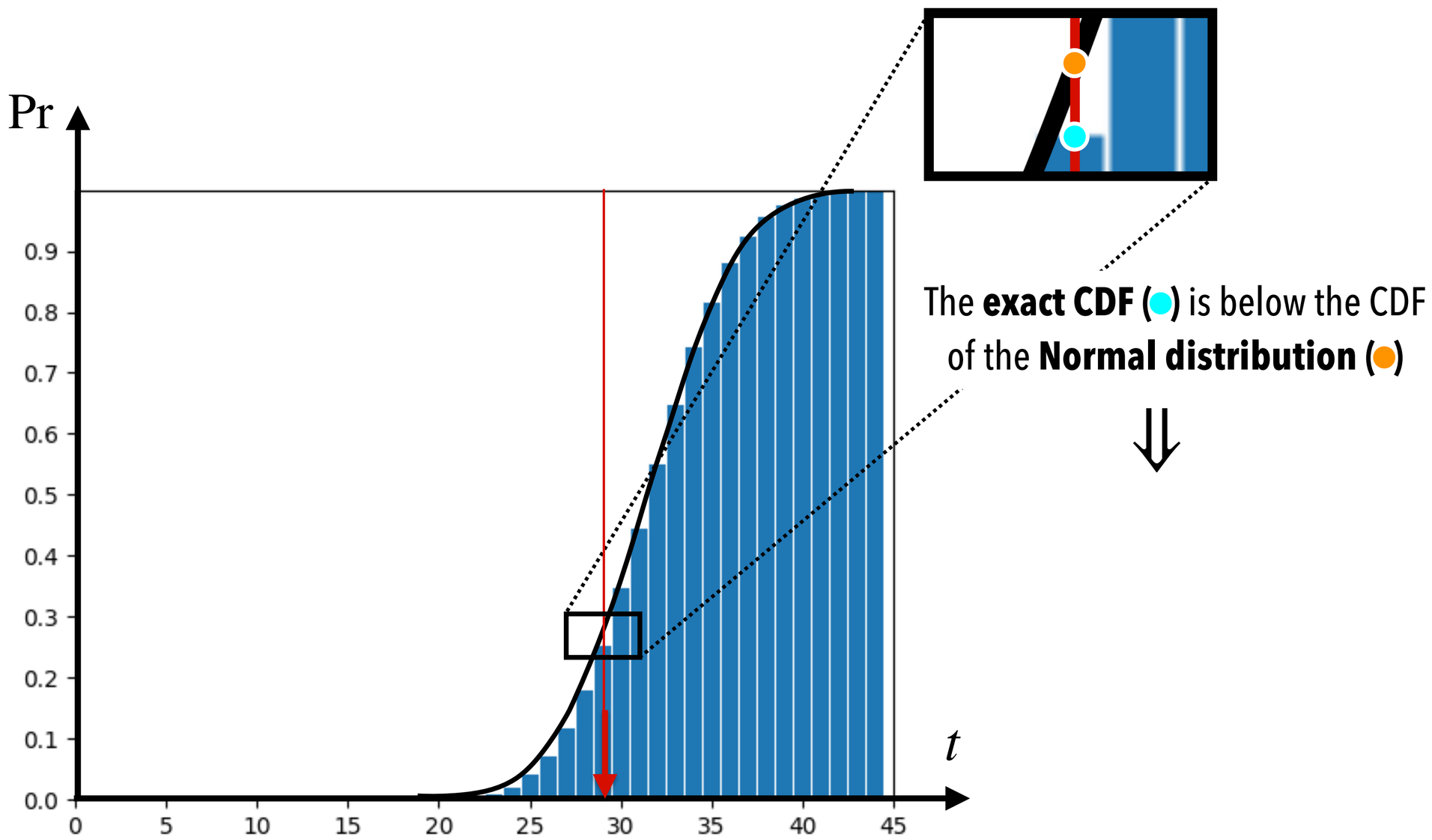
IS THE CDF OF NORMAL DISTRIBUTION SAFE?

*The CDF of the normal distribution is **not safe** for analysis of real-time systems.*



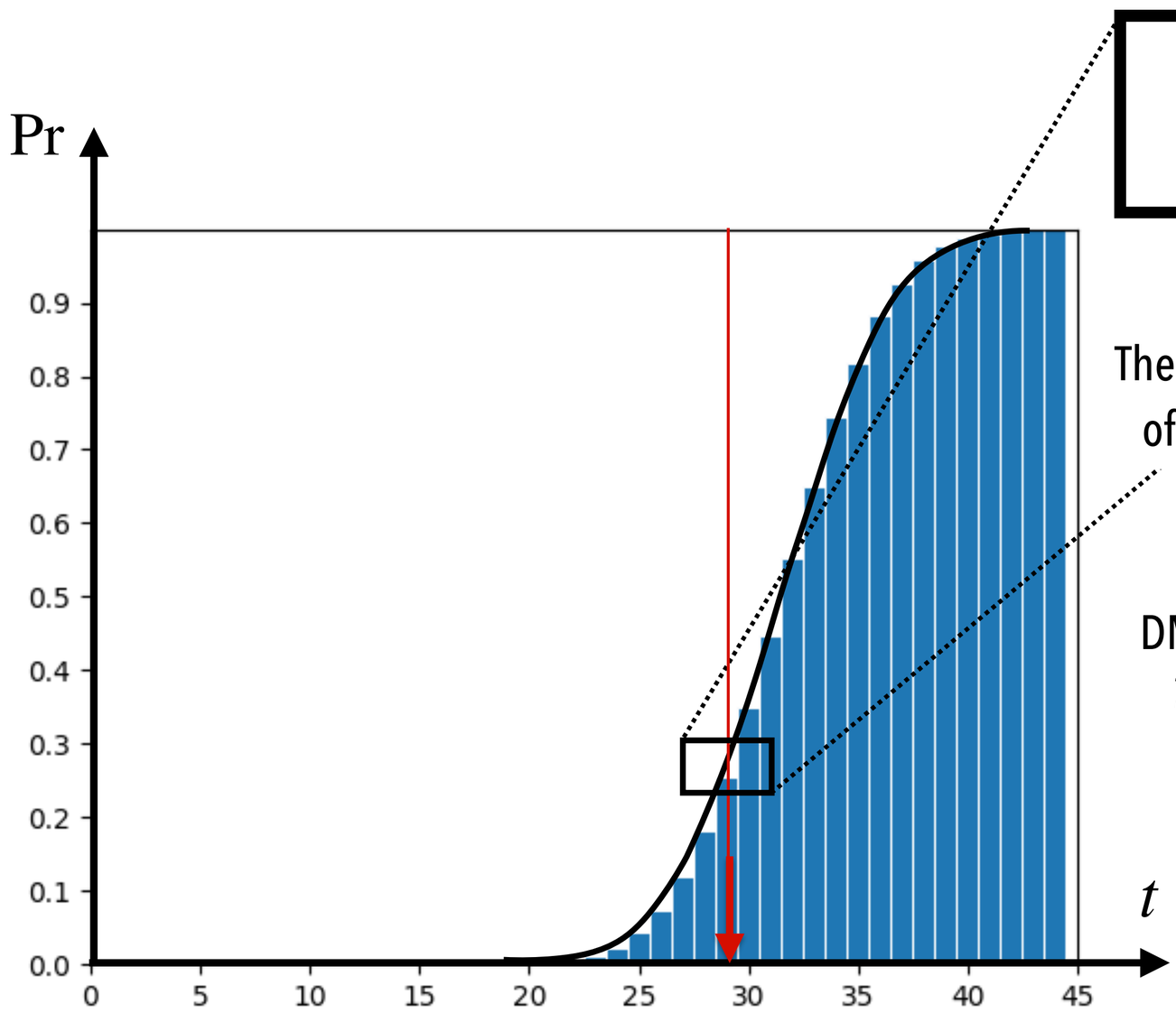
IS THE CDF OF NORMAL DISTRIBUTION SAFE?

*The CDF of the normal distribution is **not safe** for analysis of real-time systems.*



IS THE CDF OF NORMAL DISTRIBUTION SAFE?

*The CDF of the normal distribution is **not safe** for analysis of real-time systems.*



The **exact CDF (●)** is below the CDF of the **Normal distribution (●)**

⇓

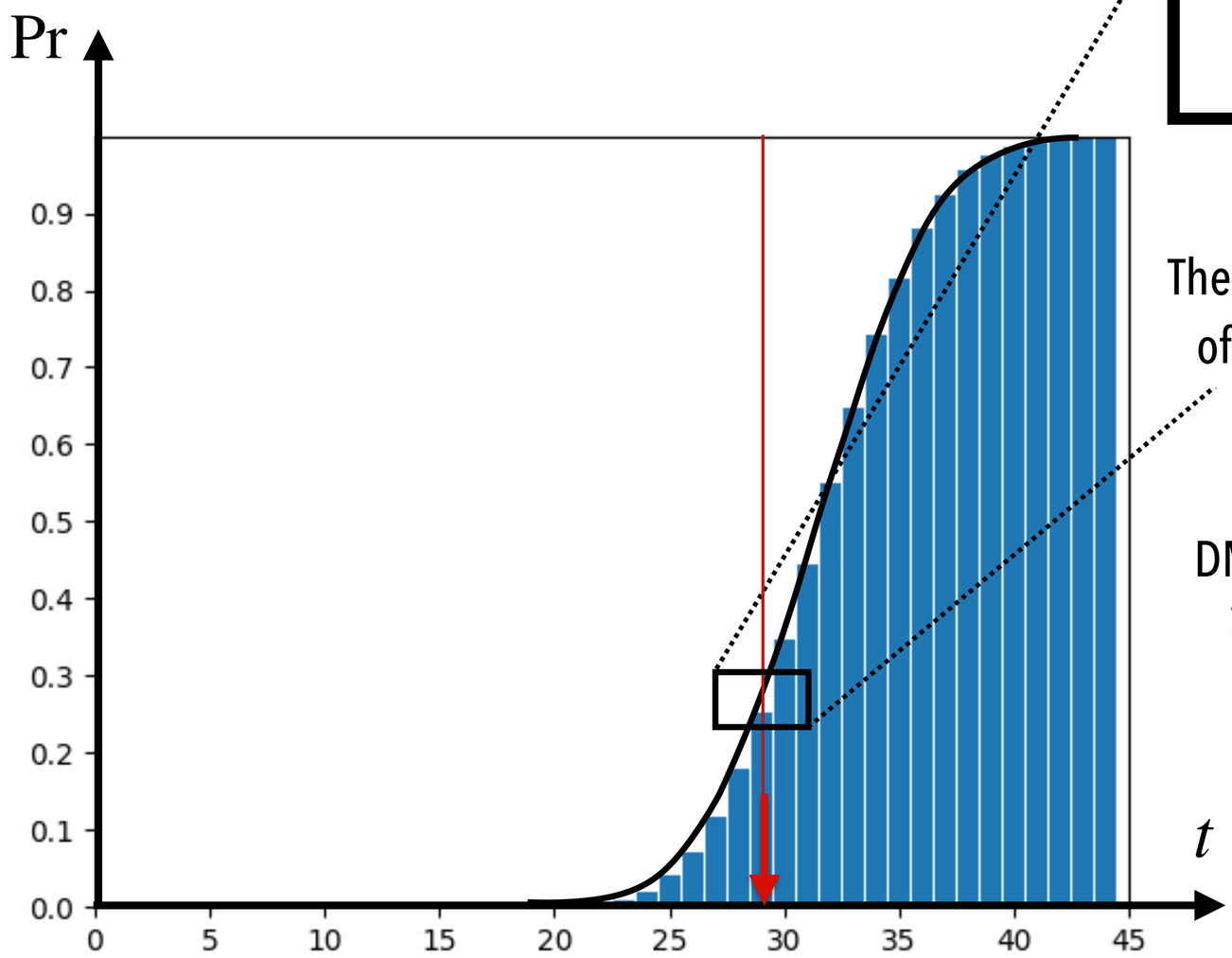
DMP(**Normal**) < DMP(**exact**)

*The probability of a bad event is **under-approximated**.*

*DMP stands for the deadline-miss probability

IS THE CDF OF NORMAL DISTRIBUTION SAFE?

*The CDF of the normal distribution is **not safe** for analysis of real-time systems.*



The **exact CDF (●)** is below the CDF of the **Normal distribution (●)**

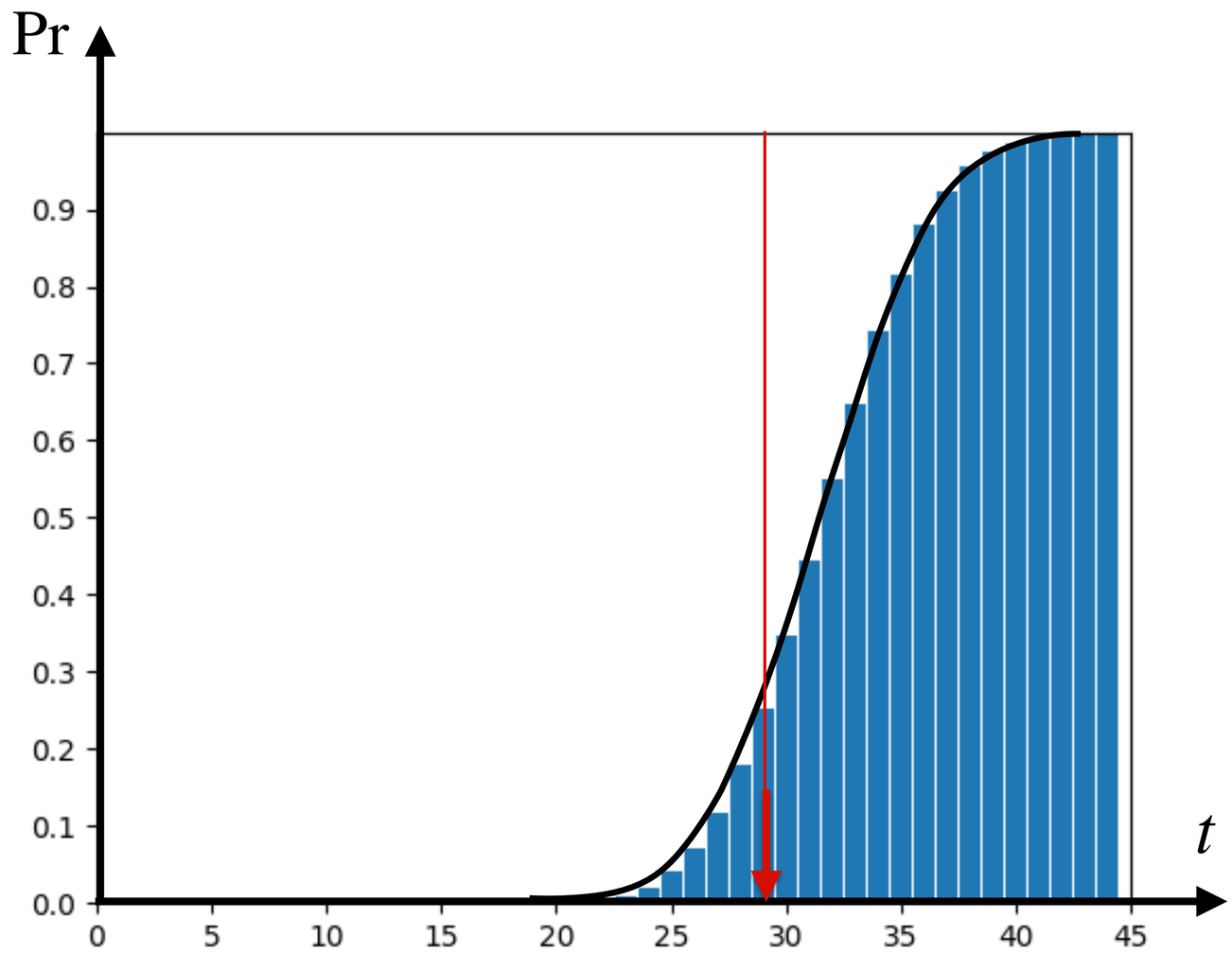
⇓

DMP(**Normal**) < DMP(**exact**)

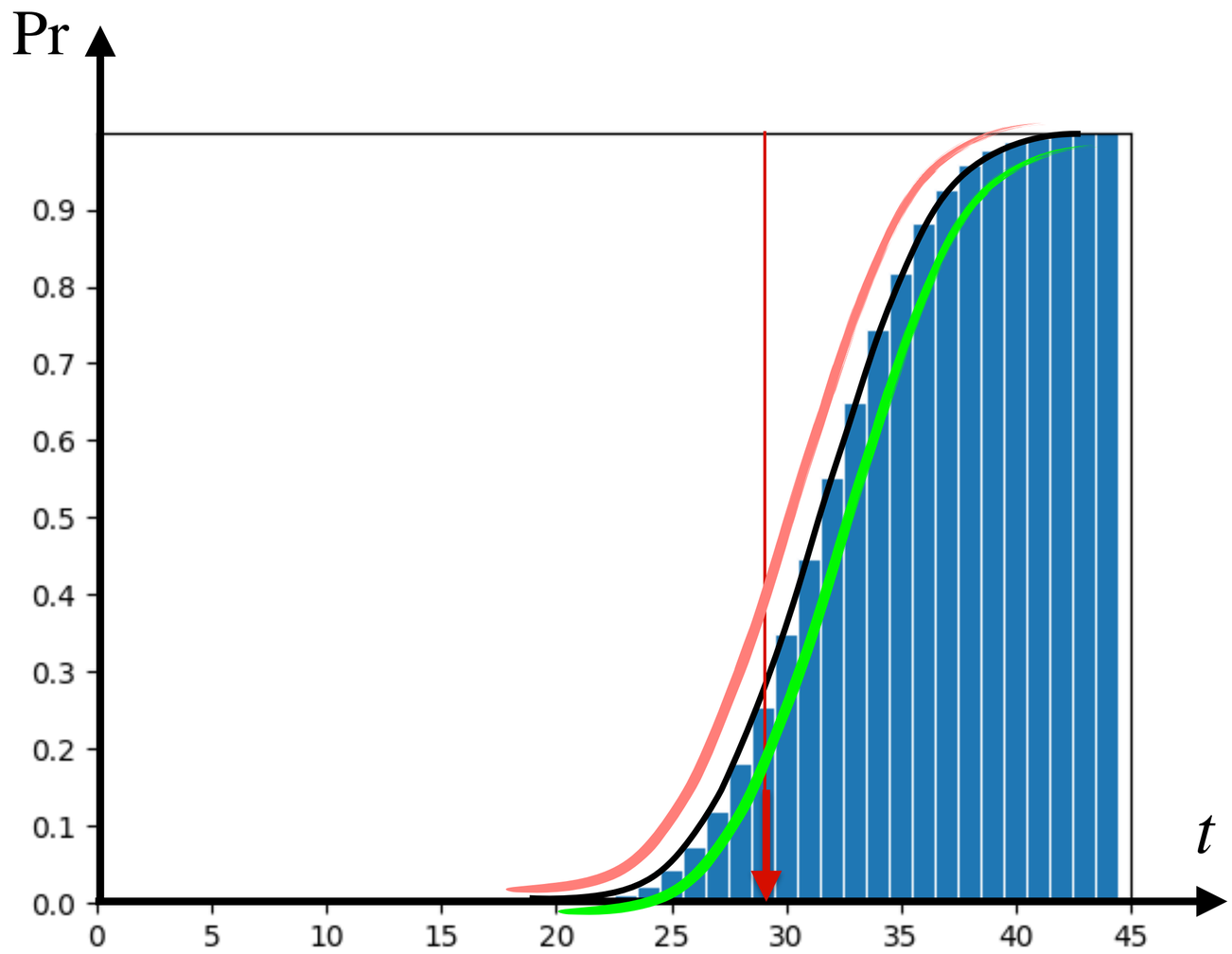
*The probability of a bad event is **under-approximated**.*

*DMP stands for the deadline-miss probability

CAN WE PROVIDE A SAFE SOLUTION?

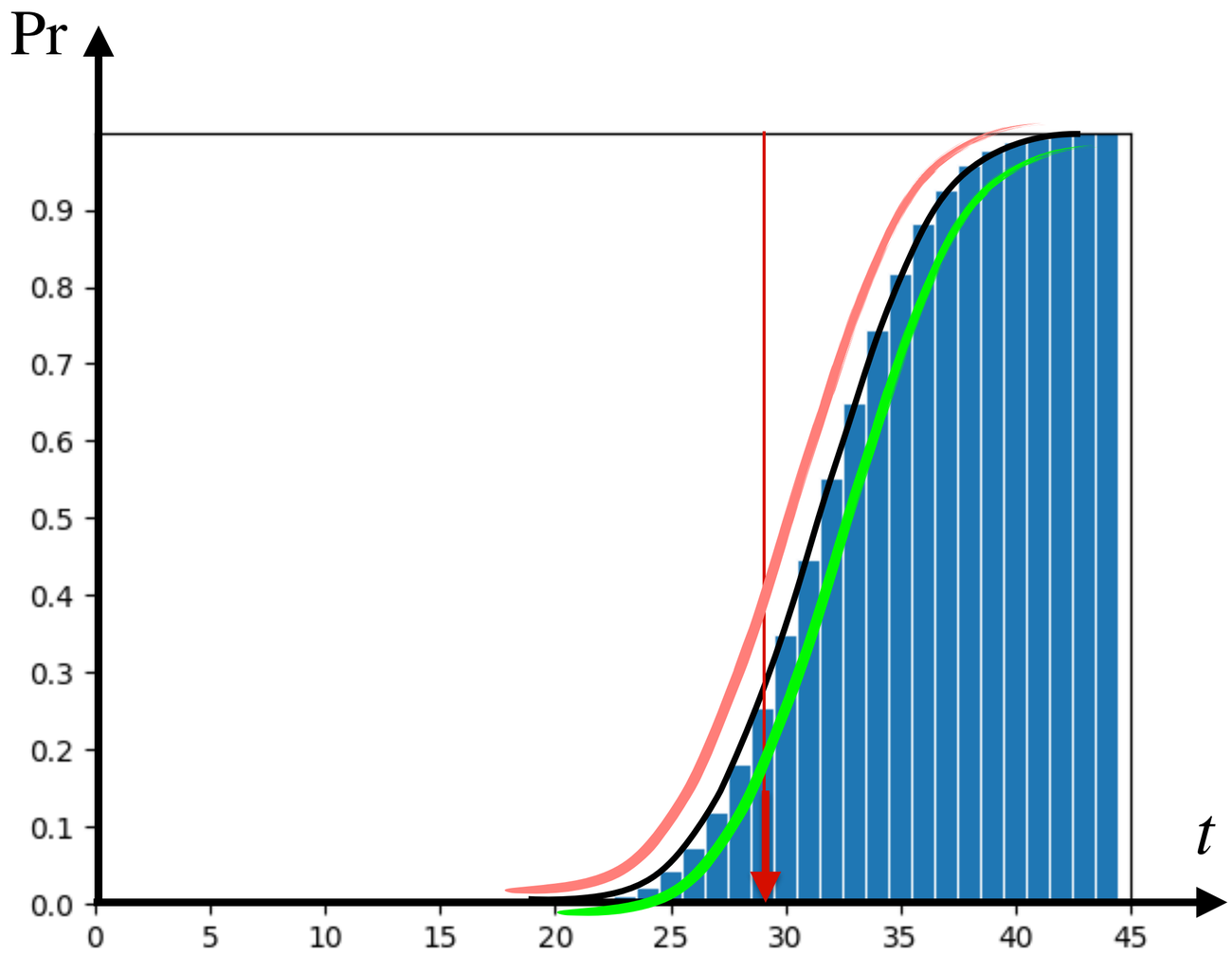


CAN WE PROVIDE A SAFE SOLUTION?



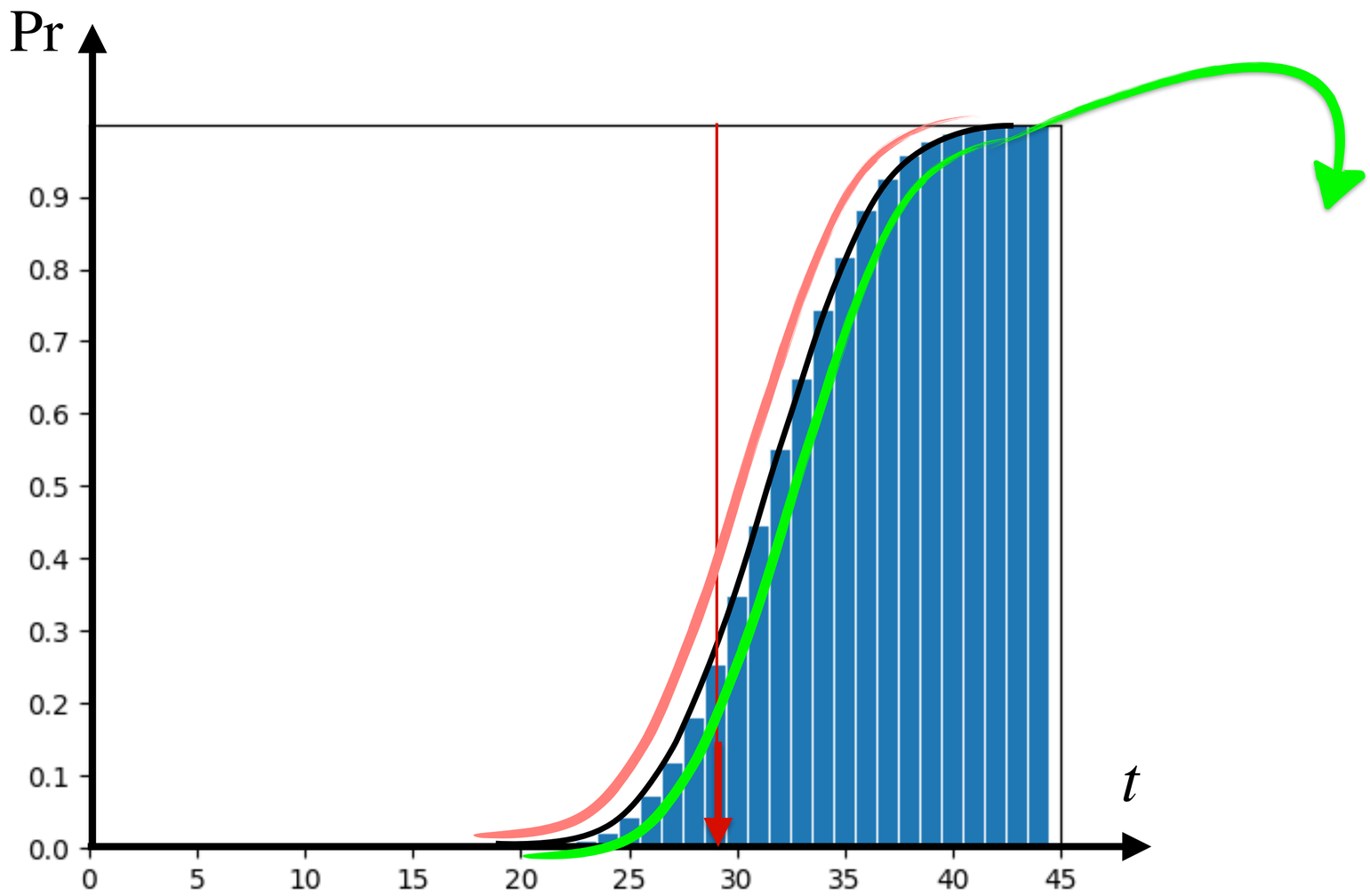
CAN WE PROVIDE A SAFE SOLUTION?

*Yes, we can use the **Berry-Esseen theorem** and quantify the **rate of convergence** towards the **Normal distribution**.*



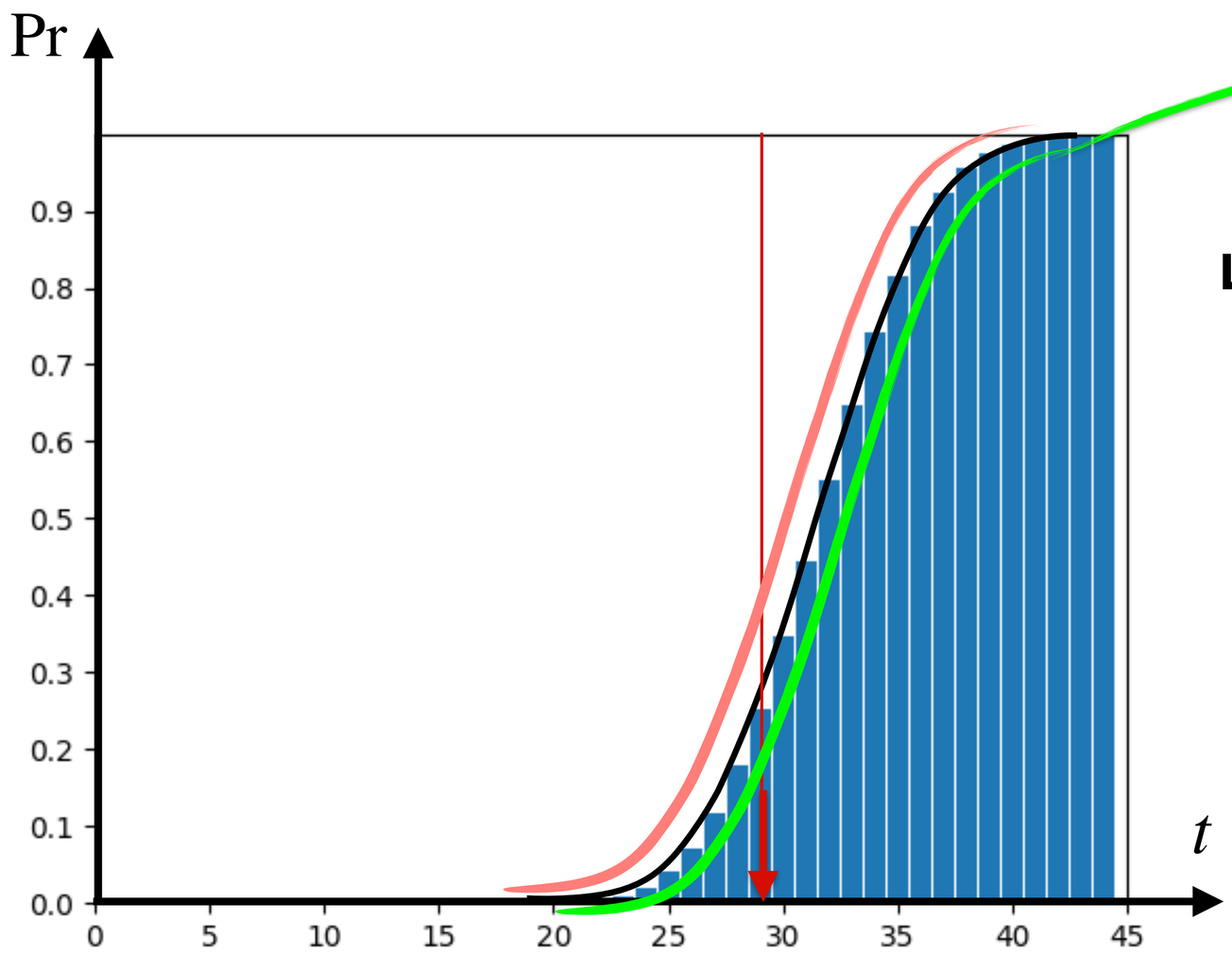
CAN WE PROVIDE A SAFE SOLUTION?

*Yes, we can use the **Berry-Esseen theorem** and quantify the **rate of convergence** towards the **Normal distribution**.*



CAN WE PROVIDE A SAFE SOLUTION?

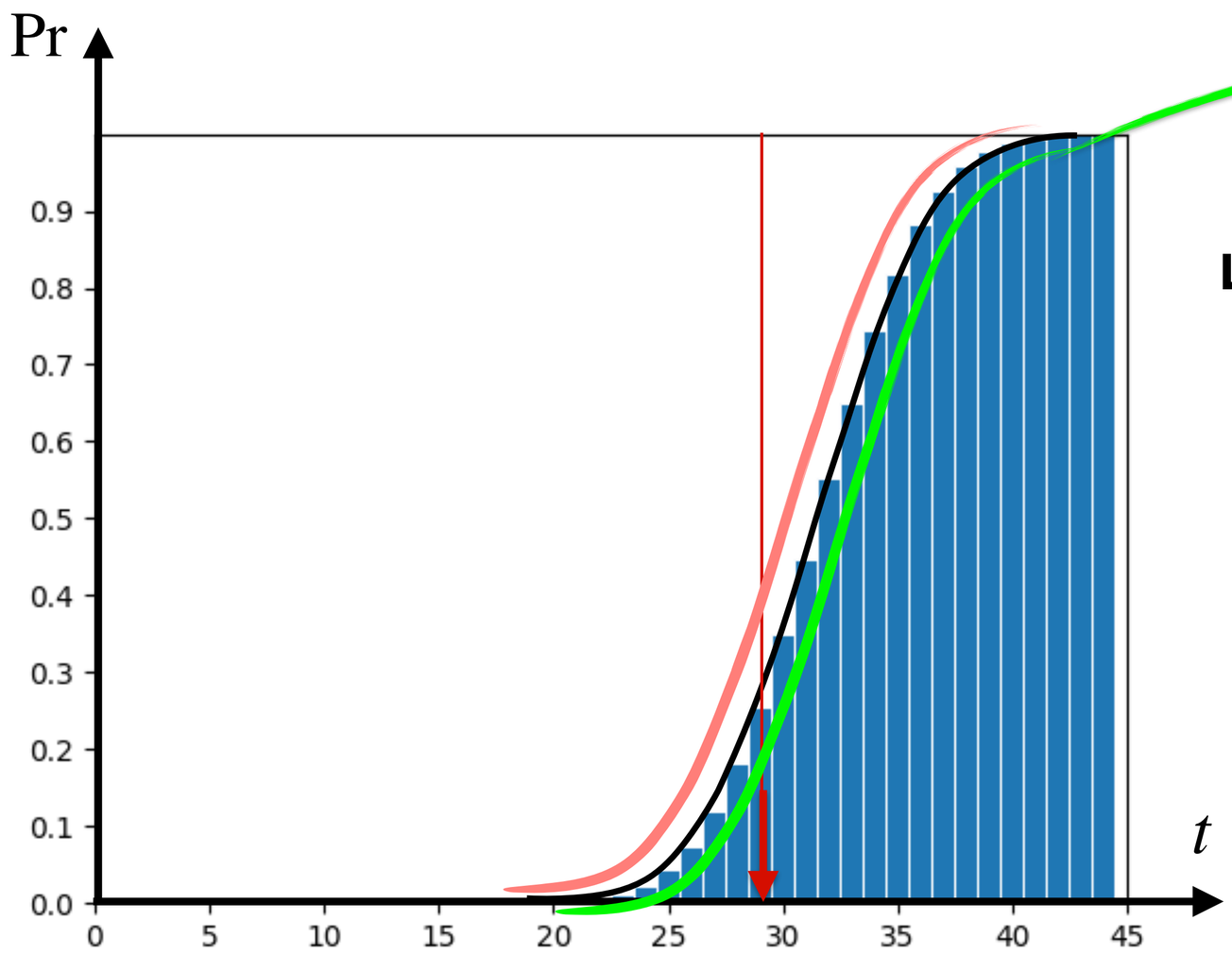
*Yes, we can use the **Berry-Esseen theorem** and quantify the **rate of convergence** towards the **Normal distribution**.*



Lower-Bound (LB) on the exact CDF distribution

CAN WE PROVIDE A SAFE SOLUTION?

*Yes, we can use the **Berry-Esseen theorem** and quantify the **rate of convergence** towards the **Normal distribution**.*

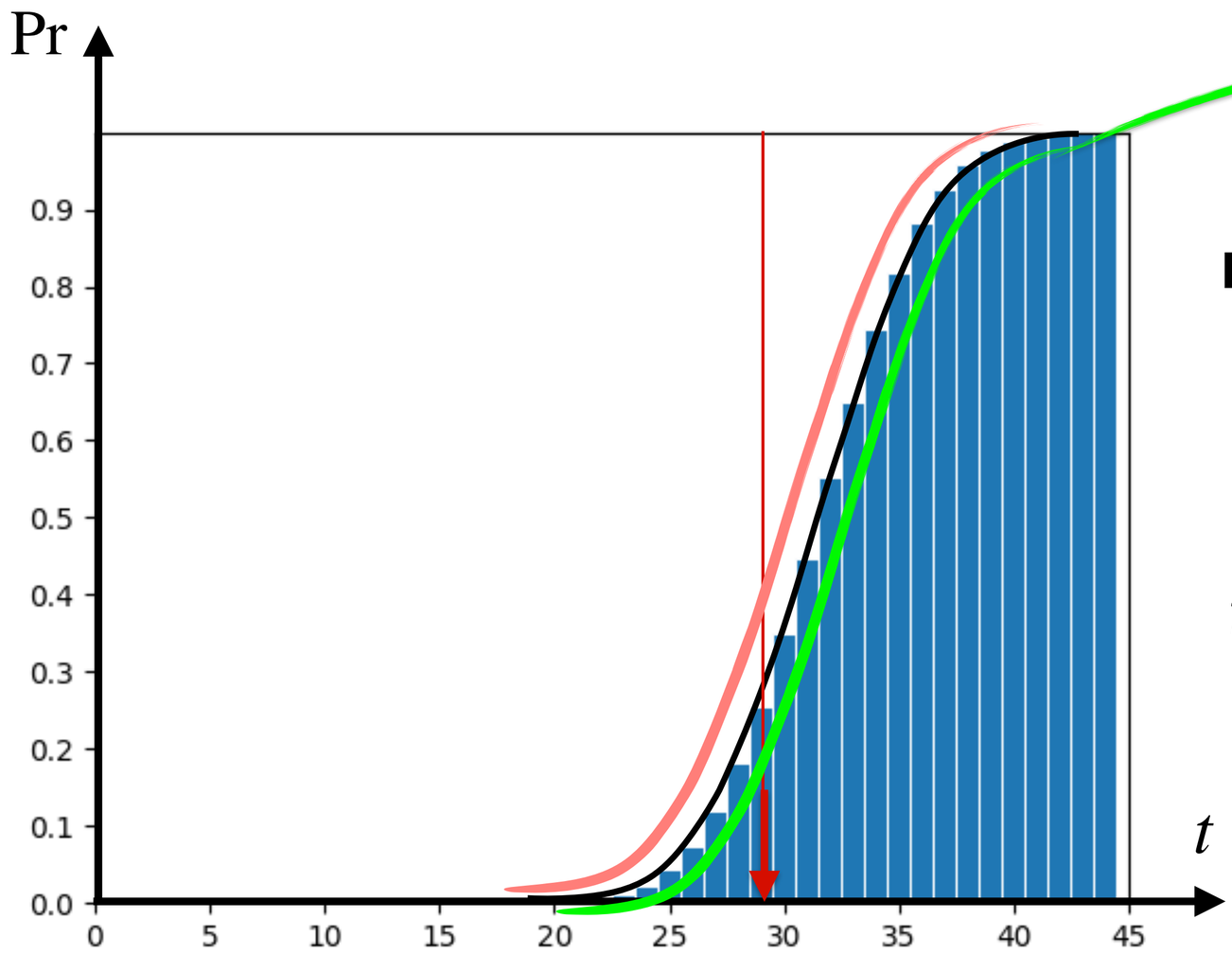


Lower-Bound (LB) on the exact CDF distribution

⇓

CAN WE PROVIDE A SAFE SOLUTION?

*Yes, we can use the **Berry-Esseen theorem** and quantify the **rate of convergence** towards the **Normal distribution**.*



Lower-Bound (LB) on the exact CDF distribution

⇓

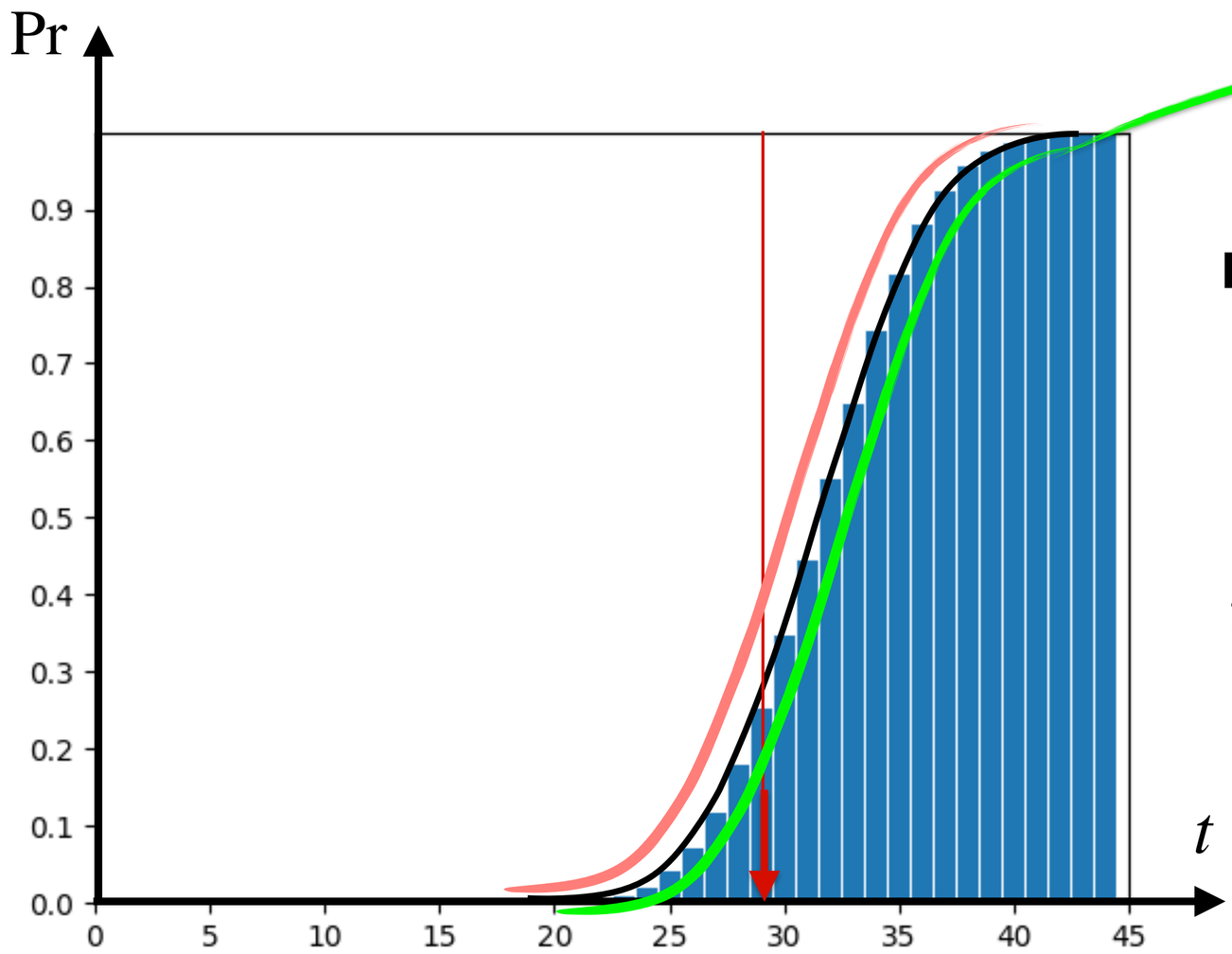
DMP(**LB**) ≥ DMP(**exact**)

*The probability of a bad event is **over-approximated**.*

*DMP stands for the deadline-miss probability

CAN WE PROVIDE A SAFE SOLUTION?

*Yes, we can use the **Berry-Esseen theorem** and quantify the **rate of convergence** towards the **Normal distribution**.*



Lower-Bound (LB) on the exact CDF distribution

⇓

DMP(**LB**) ≥ DMP(**exact**)

*The probability of a bad event is **over-approximated**.*

*DMP stands for the deadline-miss probability

EVALUATION

EVALUATION

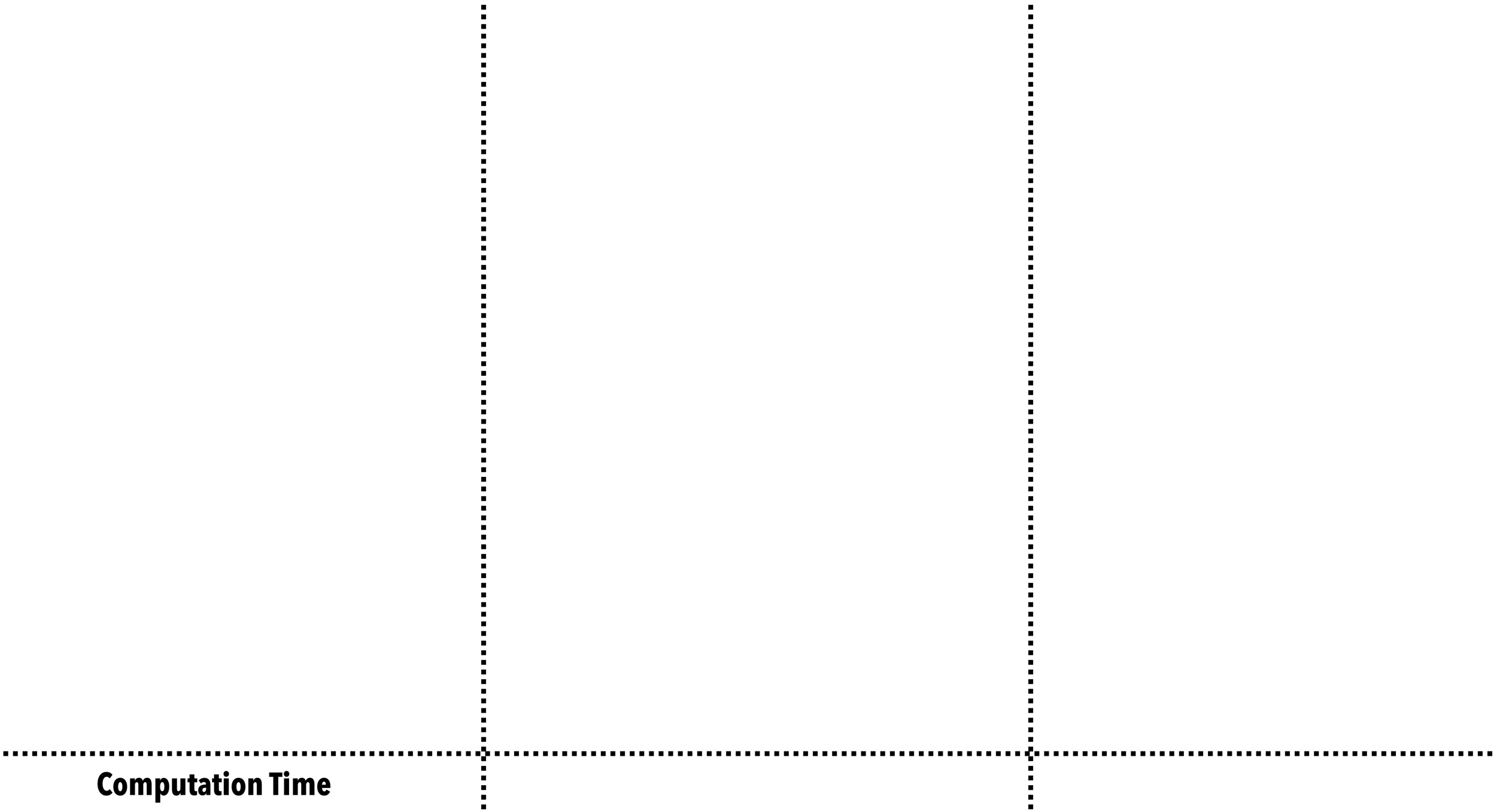
*What are the benefits of using the **proposed approach** over the most efficient analysis (**circular convolution**) for deriving the **exact distribution**?*

EVALUATION

*What are the benefits of using the **proposed approach** over the most efficient analysis (**circular convolution**) for deriving the **exact distribution**?*

EVALUATION

*What are the benefits of using the **proposed approach** over the most efficient analysis (**circular convolution**) for deriving the **exact distribution**?*



EVALUATION

*What are the benefits of using the **proposed approach** over the most efficient analysis (**circular convolution**) for deriving the **exact distribution**?*

Computation Time

Memory Footprint

EVALUATION

*What are the benefits of using the **proposed approach** over the most efficient analysis (**circular convolution**) for deriving the **exact distribution**?*

Computation Time

Memory Footprint

Approximation Accuracy

EVALUATION Computation Time

EVALUATION Computation Time

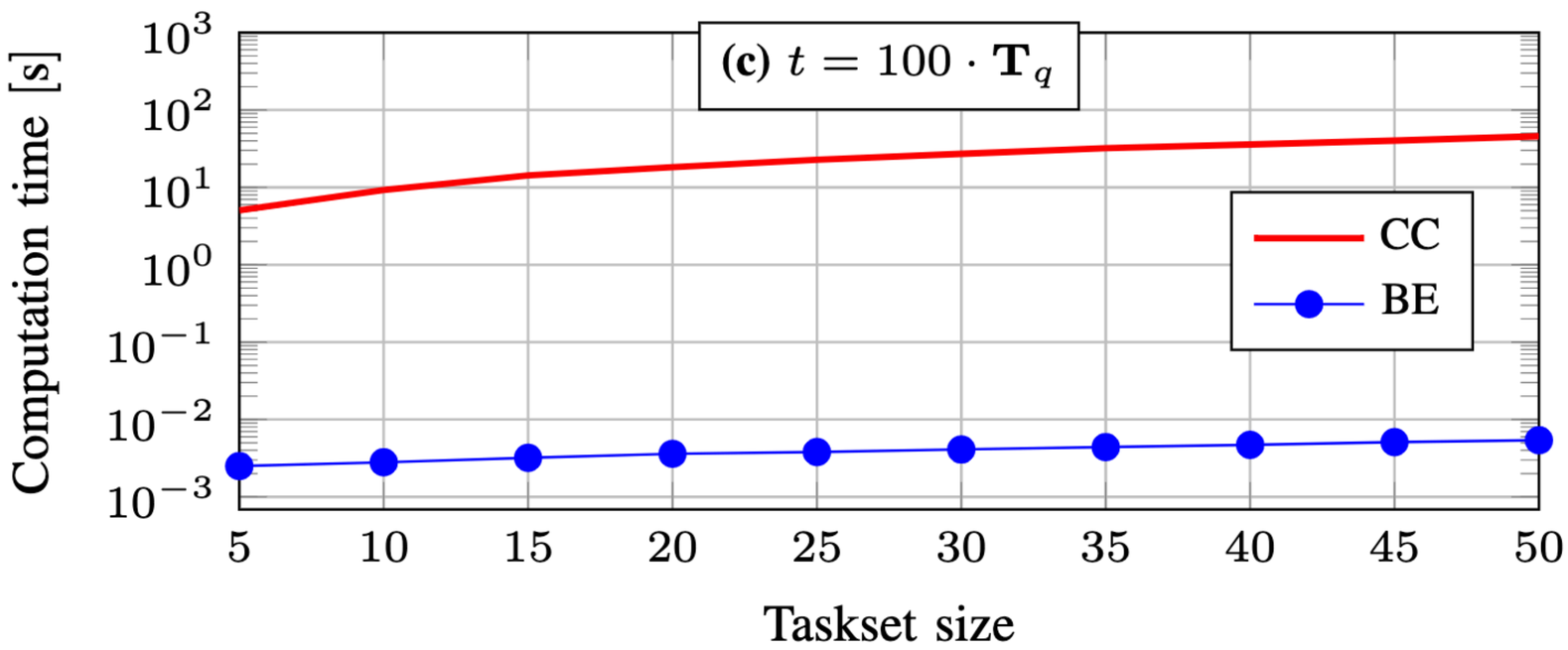
Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

EVALUATION Computation Time

Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

EVALUATION Computation Time

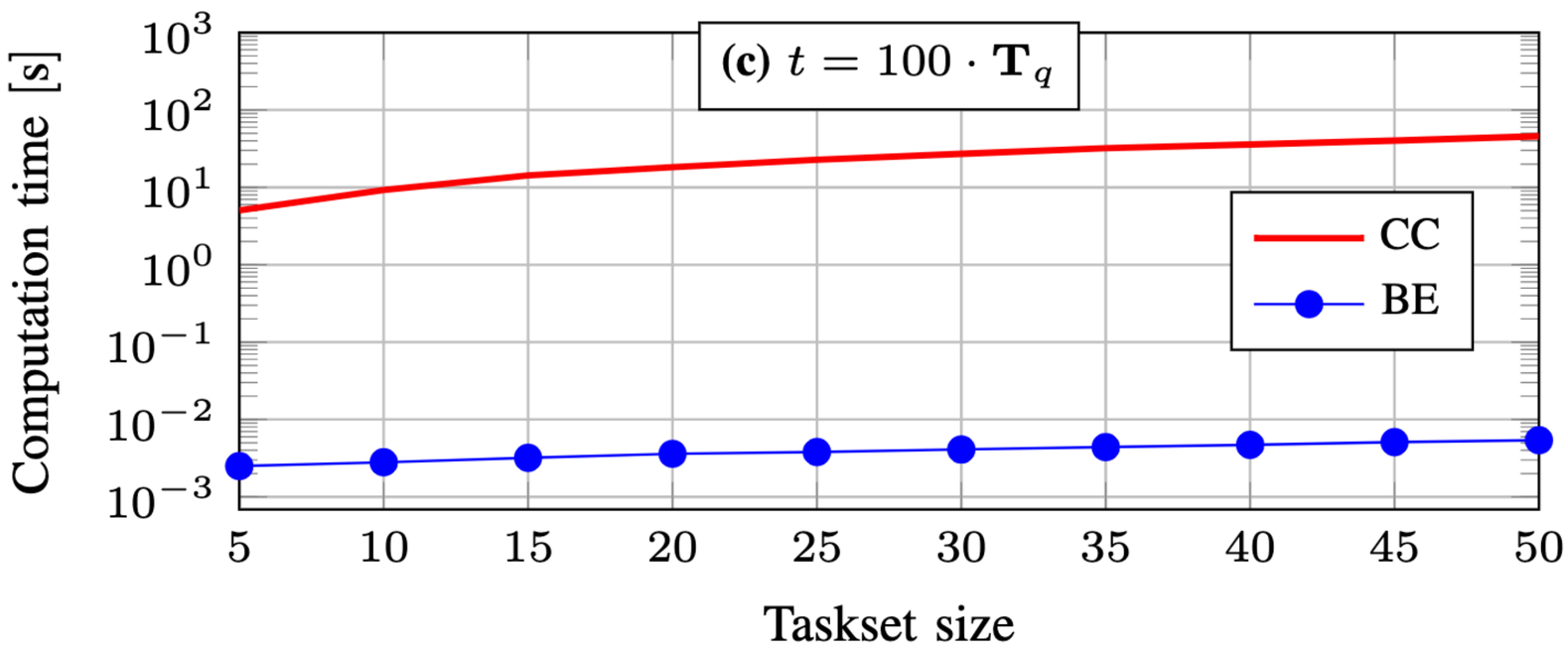


*Lower is better

Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

EVALUATION Computation Time



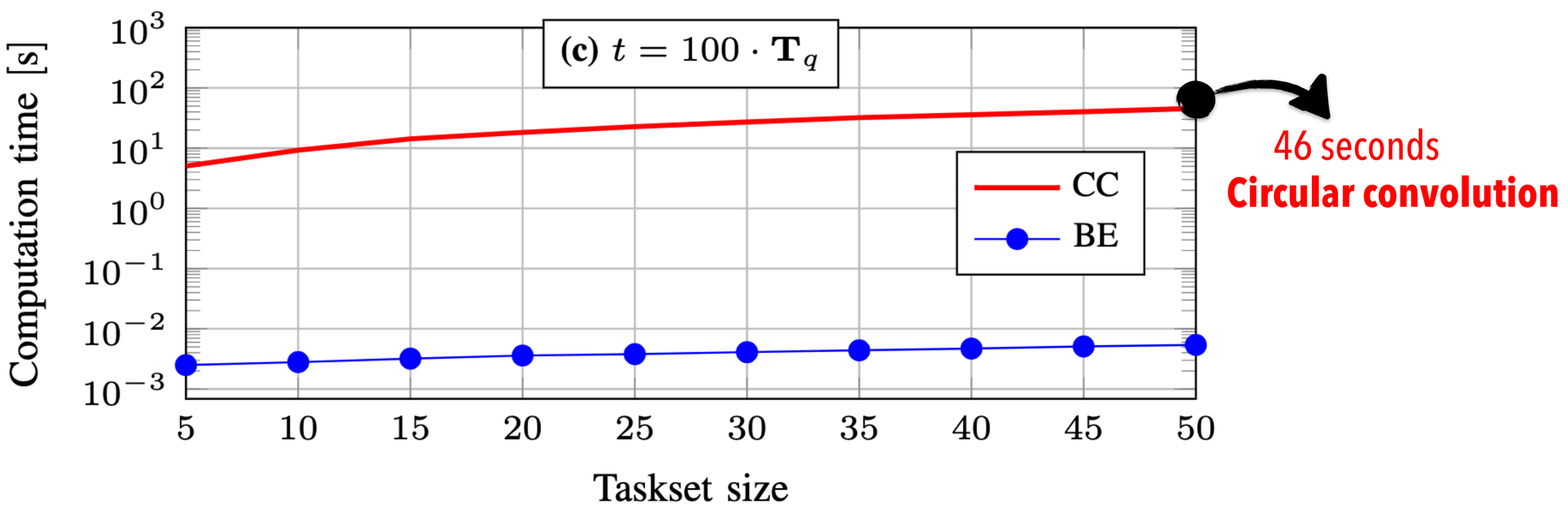
*Lower is better

Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

Comparison point: Taskset size = 50.

EVALUATION Computation Time



46 seconds
Circular convolution

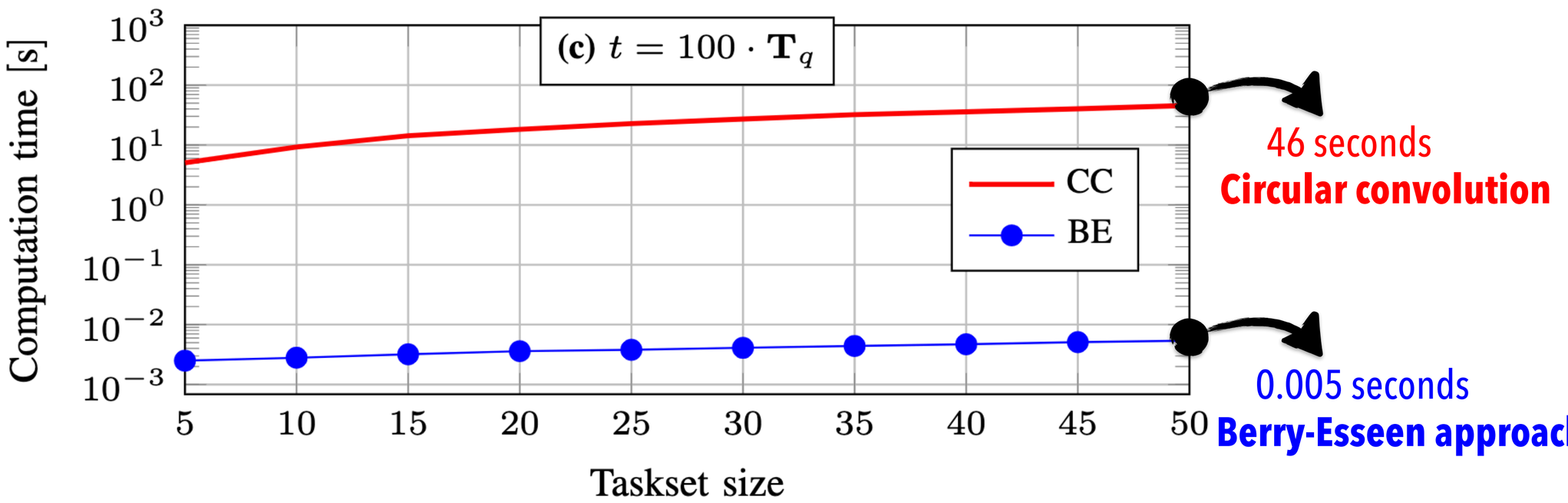
*Lower is better

Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

Comparison point: Taskset size = 50.

EVALUATION Computation Time



*Lower is better

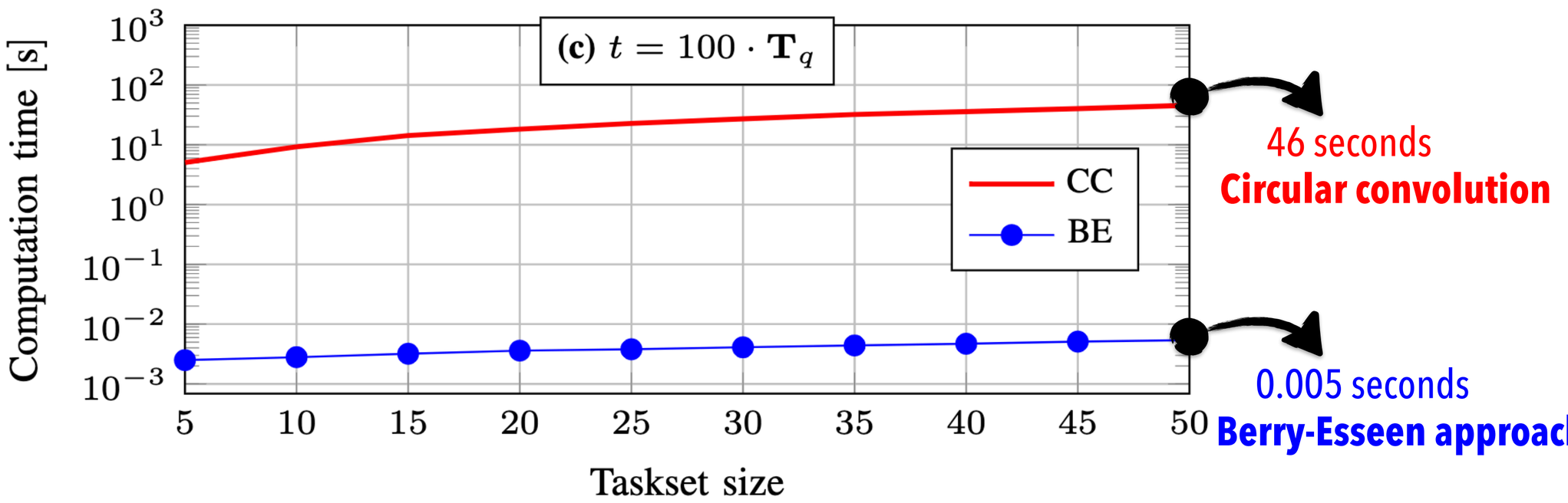
Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

Comparison point: Taskset size = 50.

EVALUATION Computation Time

*Computation time is **significantly improved** over the **circular convolution** approach.*



*Lower is better

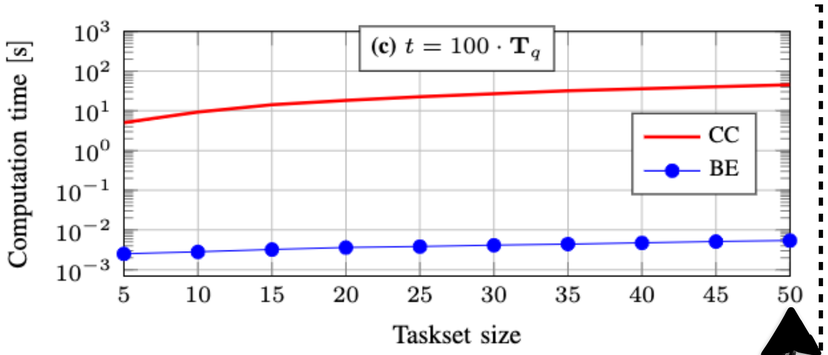
Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

Comparison point: Taskset size = 50.

EVALUATION

*What are the benefits of using the **proposed approach** over the most efficient analysis (**circular convolution**) for deriving the **exact distribution**?*



Comparison point: Taskset size = 50.

Circular Convolution (CC): 46 **seconds**

Berry-Essen (BE): 0.005 **seconds**

Computation time is **significantly improved** over the **circular convolution** approach.

Computation Time | **Memory Footprint** | **Approximation Accuracy**

EVALUATION Memory Footprint

EVALUATION Memory Footprint

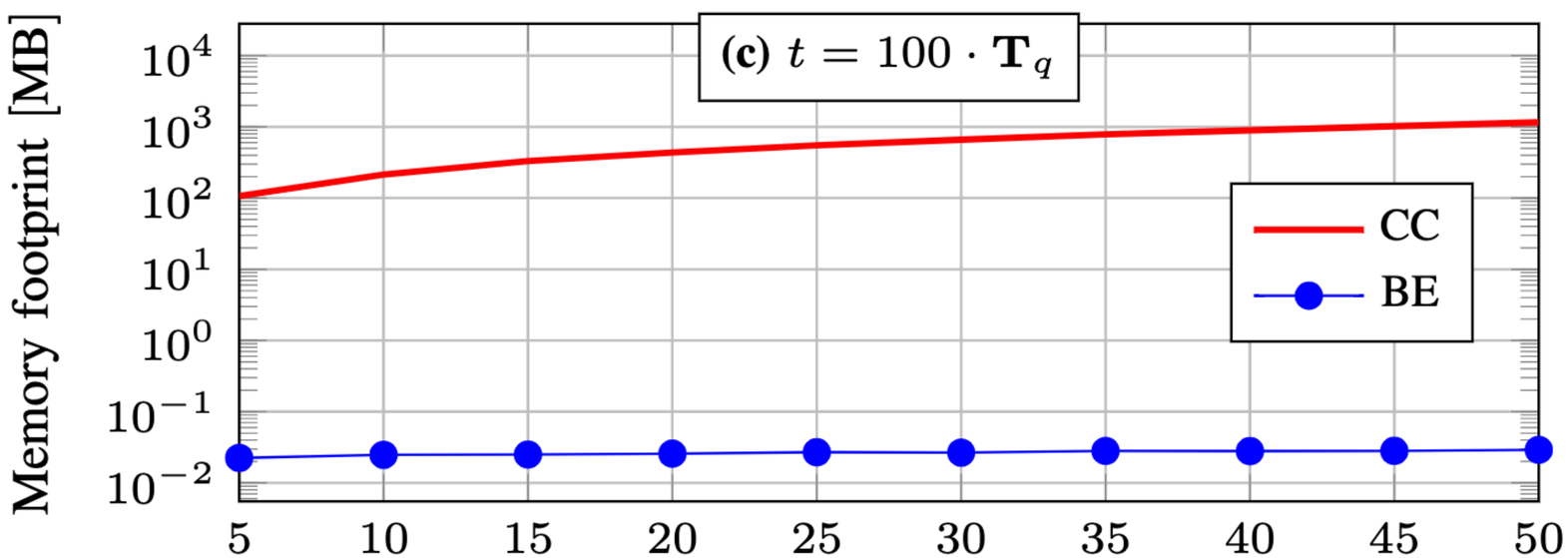
Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

EVALUATION Memory Footprint

Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

EVALUATION Memory Footprint

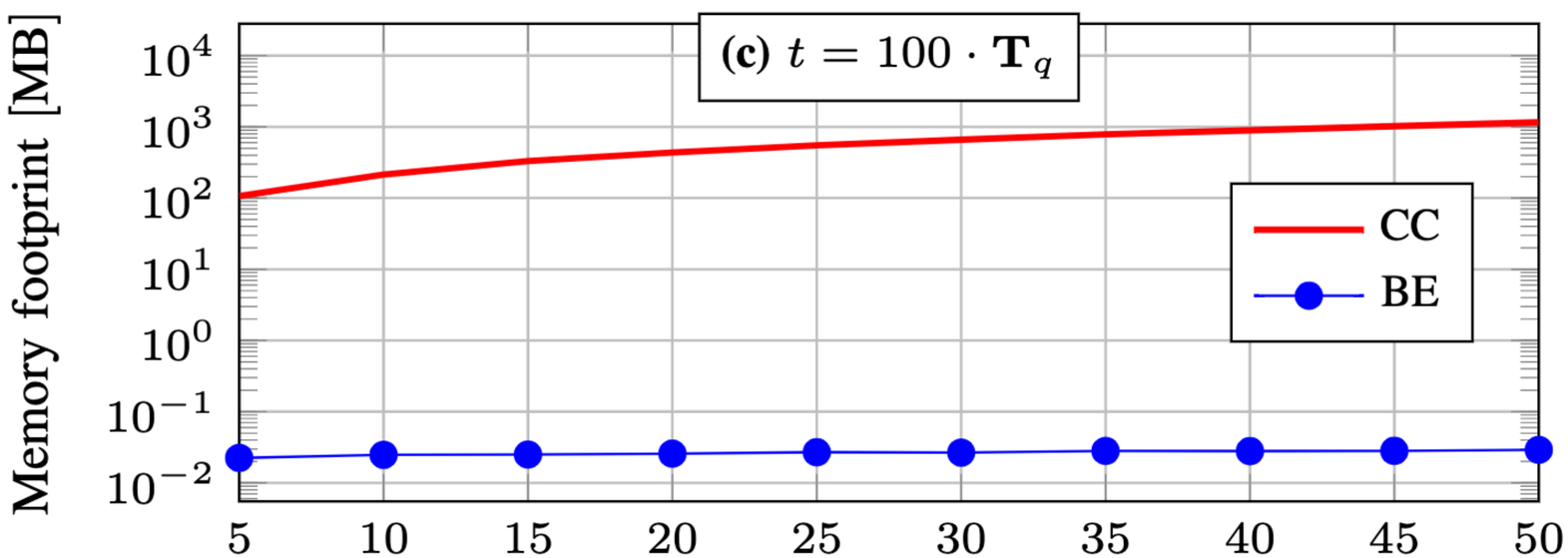


*Lower is better

Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

EVALUATION Memory Footprint



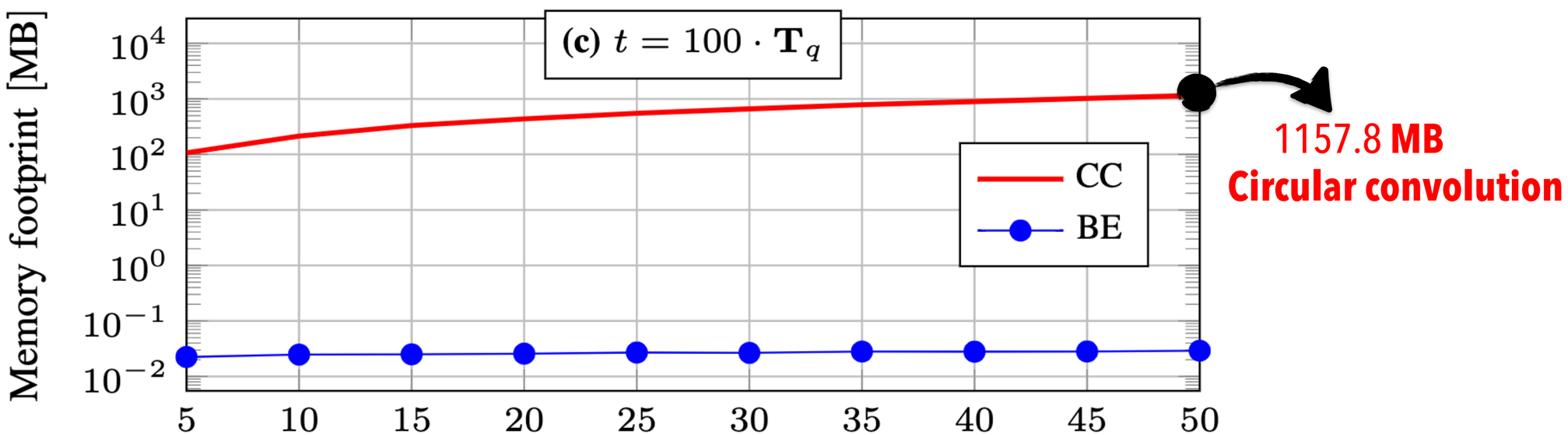
*Lower is better

Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

Comparison point: Taskset size = 50.

EVALUATION Memory Footprint



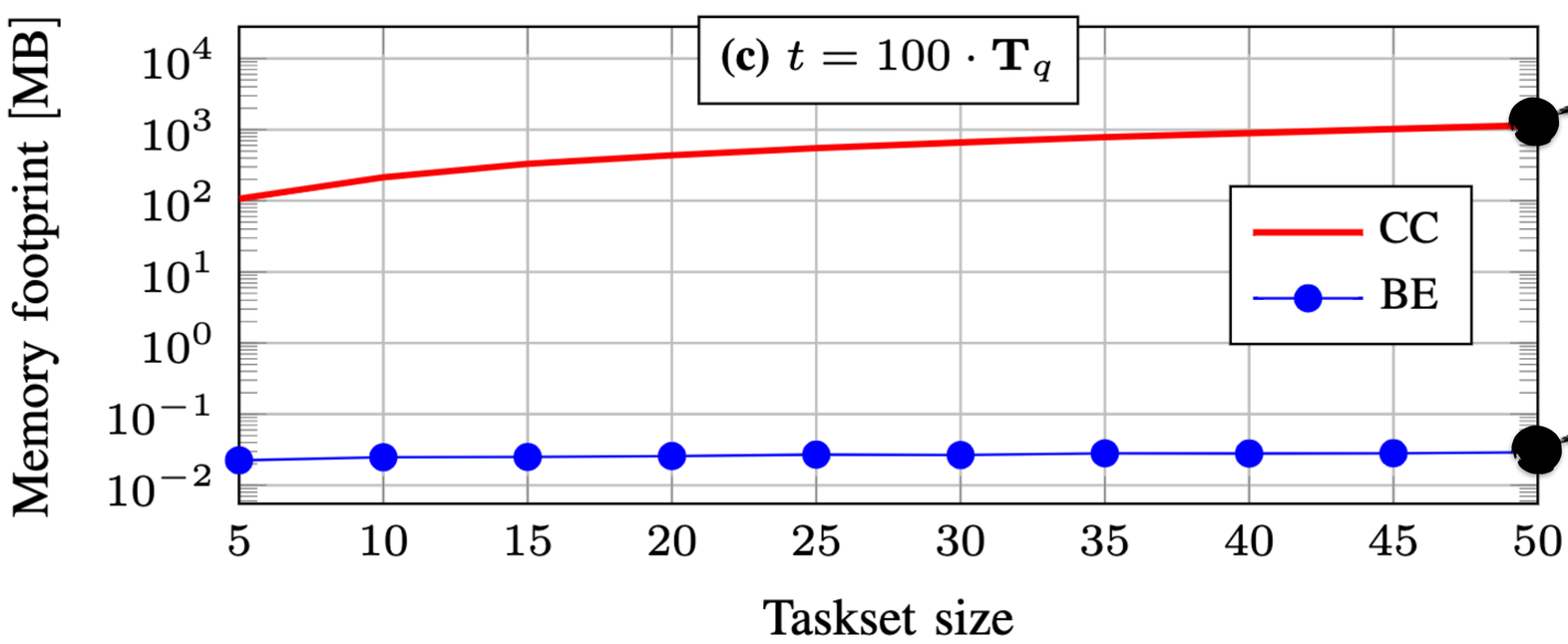
*Lower is better

Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

Comparison point: Taskset size = 50.

EVALUATION Memory Footprint



1157.8 **MB**
Circular convolution

0.029 **MB**
Berry-Esseen approach

*Lower is better

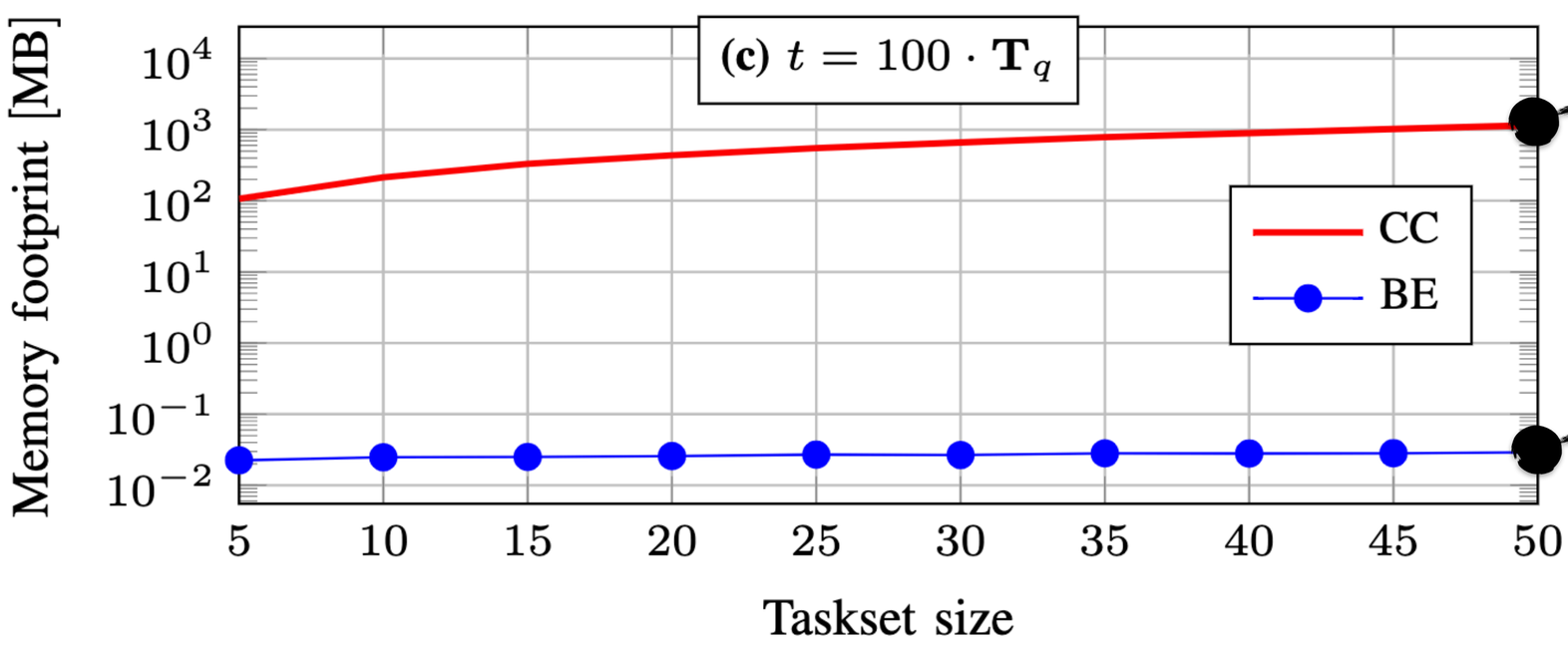
Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

Comparison point: Taskset size = 50.

EVALUATION Memory Footprint

*Memory footprint is **significantly improved** over the **circular convolution** approach.*



1157.8 **MB**
Circular convolution

0.029 **MB**
Berry-Esseen approach

*Lower is better

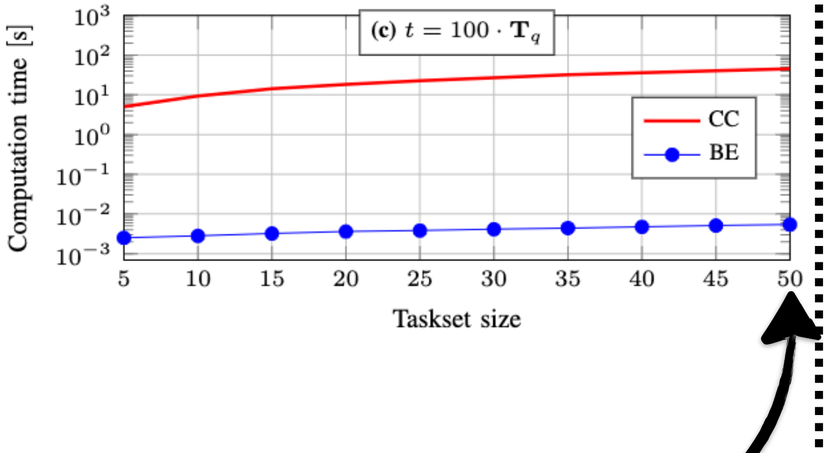
Investigated scenario*:* Analysis over 100 periods of the lowest-priority task.

Variable parameter: Taskset size, from 5 to 50 (step 5).

Comparison point: Taskset size = 50.

EVALUATION

*What are the benefits of using the **proposed approach** over the most efficient analysis (**circular convolution**) for deriving the **exact distribution**?*



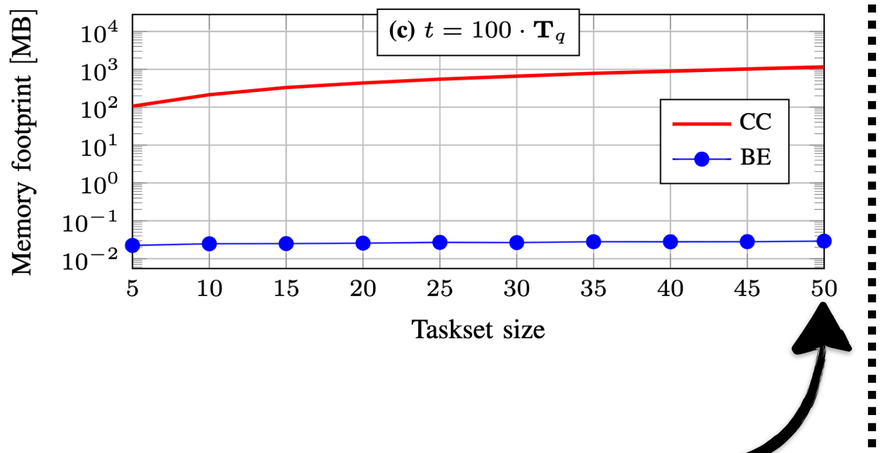
Comparison point: Taskset size = 50.

Circular Convolution (CC): 46 **seconds**

Berry-Essen (BE): 0.005 **seconds**

Computation time is **significantly improved** over the **circular convolution** approach.

Computation Time



Comparison point: Taskset size = 50.

CC: 1157.8 **MB**

BE: 0.029 **MB**

Memory footprint is **significantly improved** over the **circular convolution** approach.

Memory Footprint

Approximation Accuracy

EVALUATION Approximation Accuracy

EVALUATION Approximation Accuracy

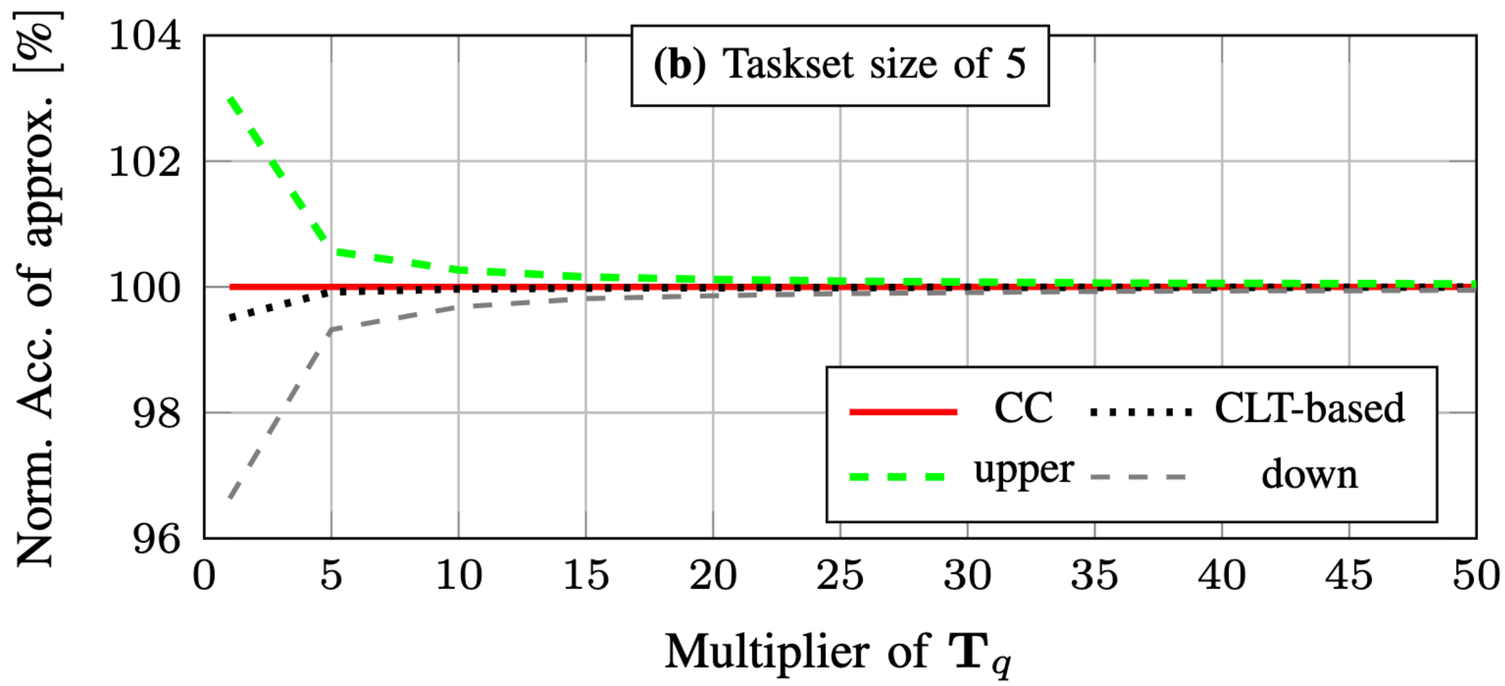
Investigated scenario: Quantile analysis over increasingly longer time intervals.

EVALUATION Approximation Accuracy

Investigated scenario: Quantile analysis over increasingly longer time intervals.

Variable parameter: Time interval under the analysis, equal to the multiplier of the period of the lowest priority task.

EVALUATION Approximation Accuracy

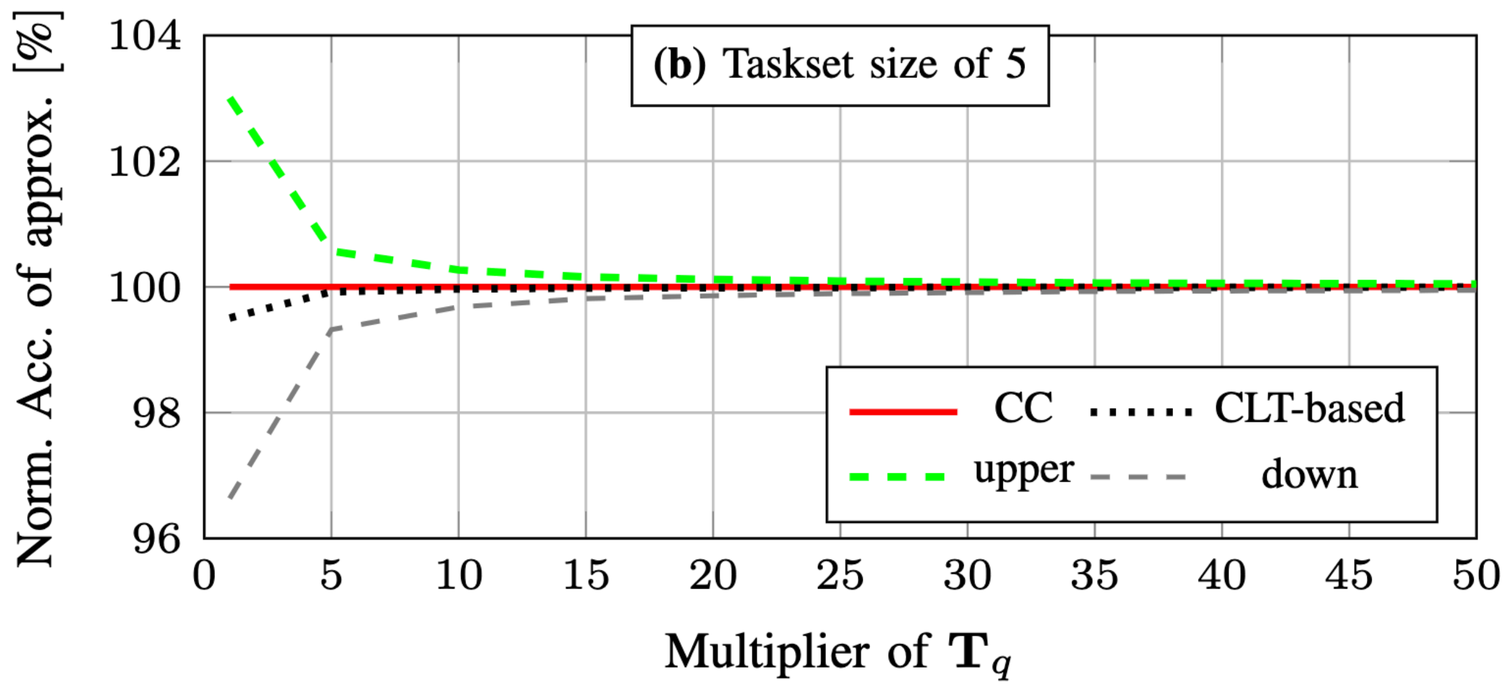


*Closer to the red value is better

Investigated scenario: Quantile analysis over increasingly longer time intervals.

Variable parameter: Time interval under the analysis, equal to the multiplier of the period of the lowest priority task.

EVALUATION Approximation Accuracy



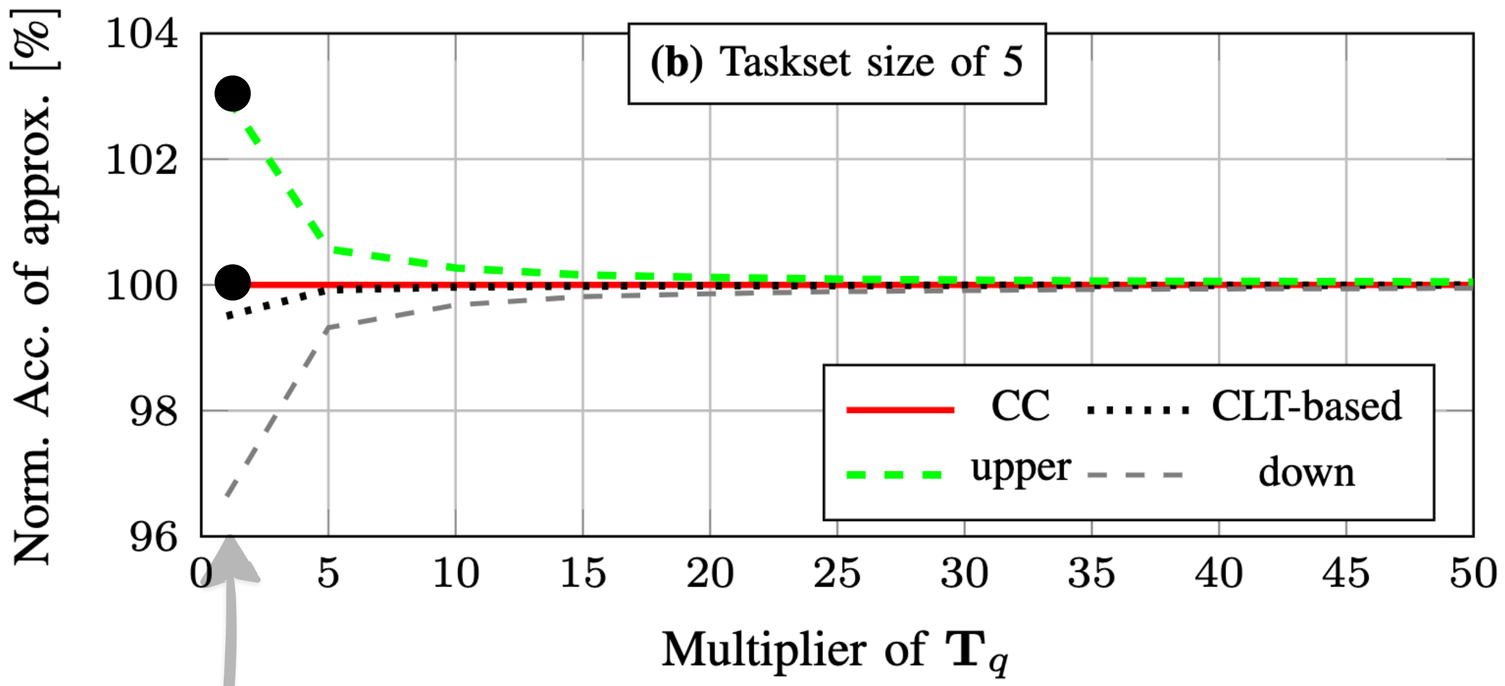
*Closer to the red value is better

Investigated scenario: Quantile analysis over increasingly longer time intervals.

Variable parameter: Time interval under the analysis, equal to the multiplier of the period of the lowest priority task.

Comparison point 1: Time interval = Period of the lowest-priority task.

EVALUATION Approximation Accuracy



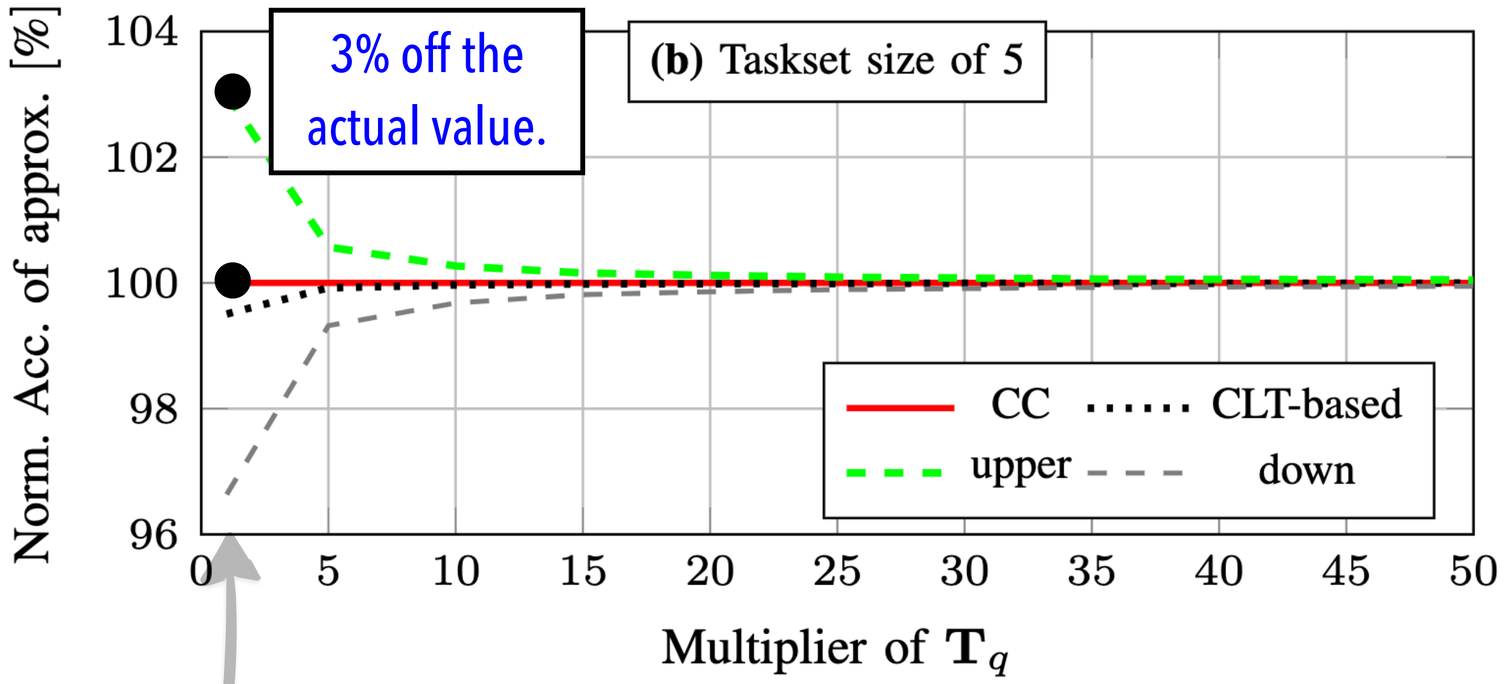
*Closer to the red value is better

Investigated scenario: Quantile analysis over increasingly longer time intervals.

Variable parameter: Time interval under the analysis, equal to the multiplier of the period of the lowest priority task.

Comparison point 1: Time interval = Period of the lowest-priority task.

EVALUATION Approximation Accuracy



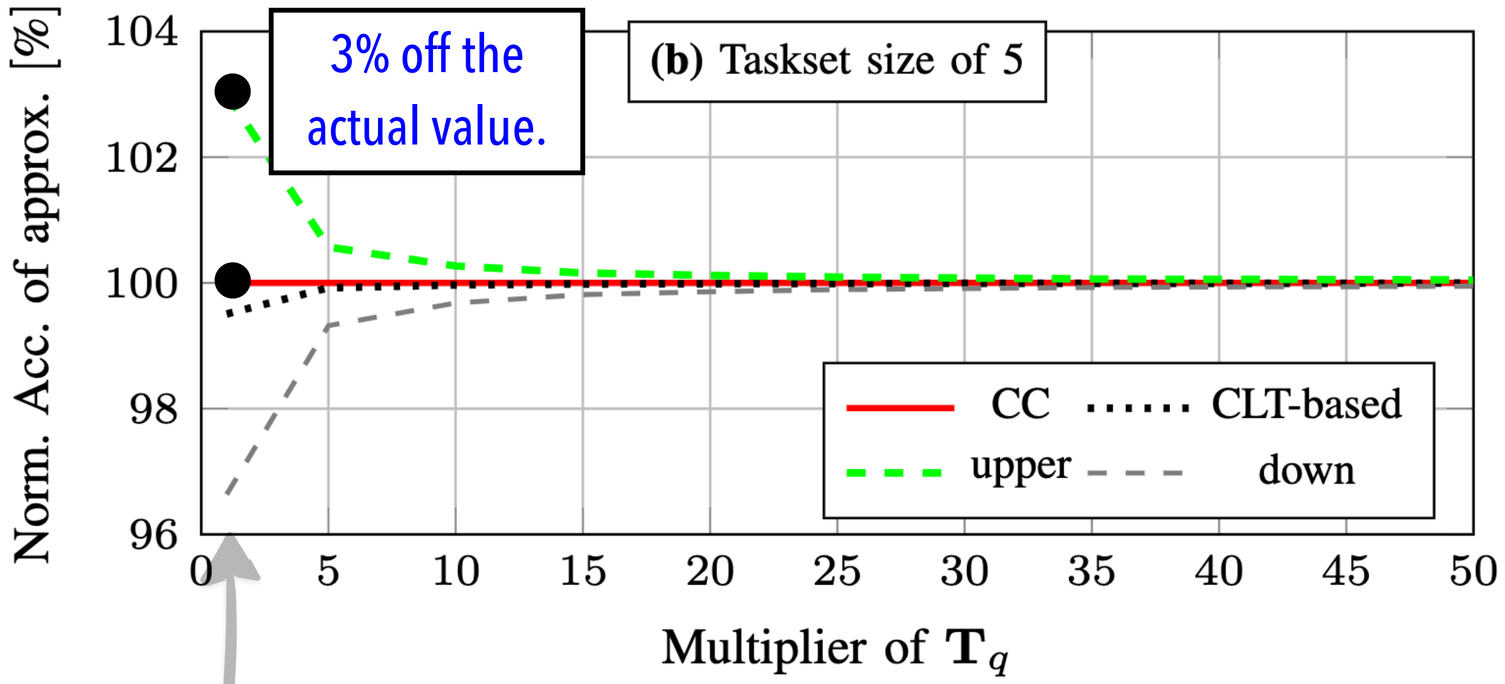
*Closer to the red value is better

Investigated scenario: Quantile analysis over increasingly longer time intervals.

Variable parameter: Time interval under the analysis, equal to the multiplier of the period of the lowest priority task.

Comparison point 1: Time interval = Period of the lowest-priority task.

EVALUATION Approximation Accuracy



*Closer to the red value is better

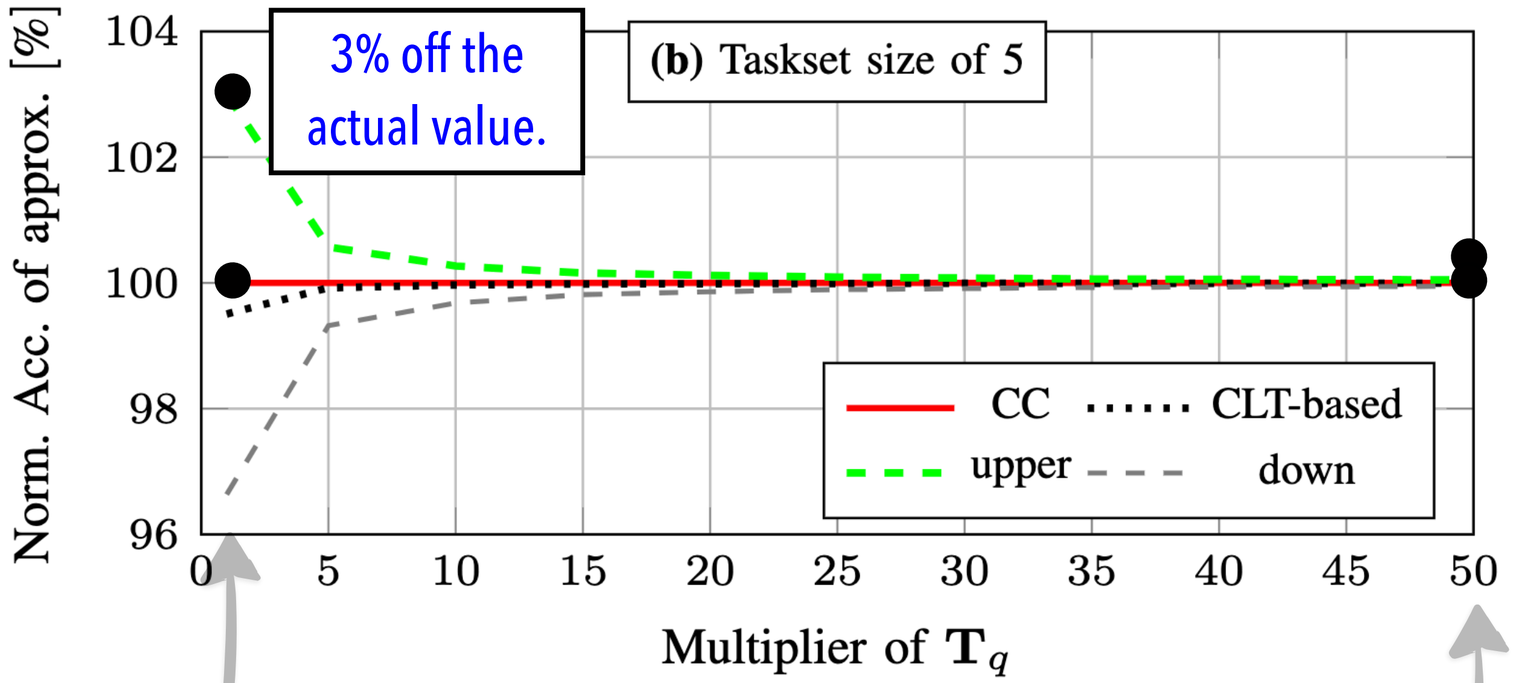
Investigated scenario: Quantile analysis over increasingly longer time intervals.

Variable parameter: Time interval under the analysis, equal to the multiplier of the period of the lowest priority task.

Comparison point 1: Time interval = Period of the lowest-priority task.

Comparison point 2: Time interval = 50 periods of the lowest-priority task.

EVALUATION Approximation Accuracy



*Closer to the red value is better

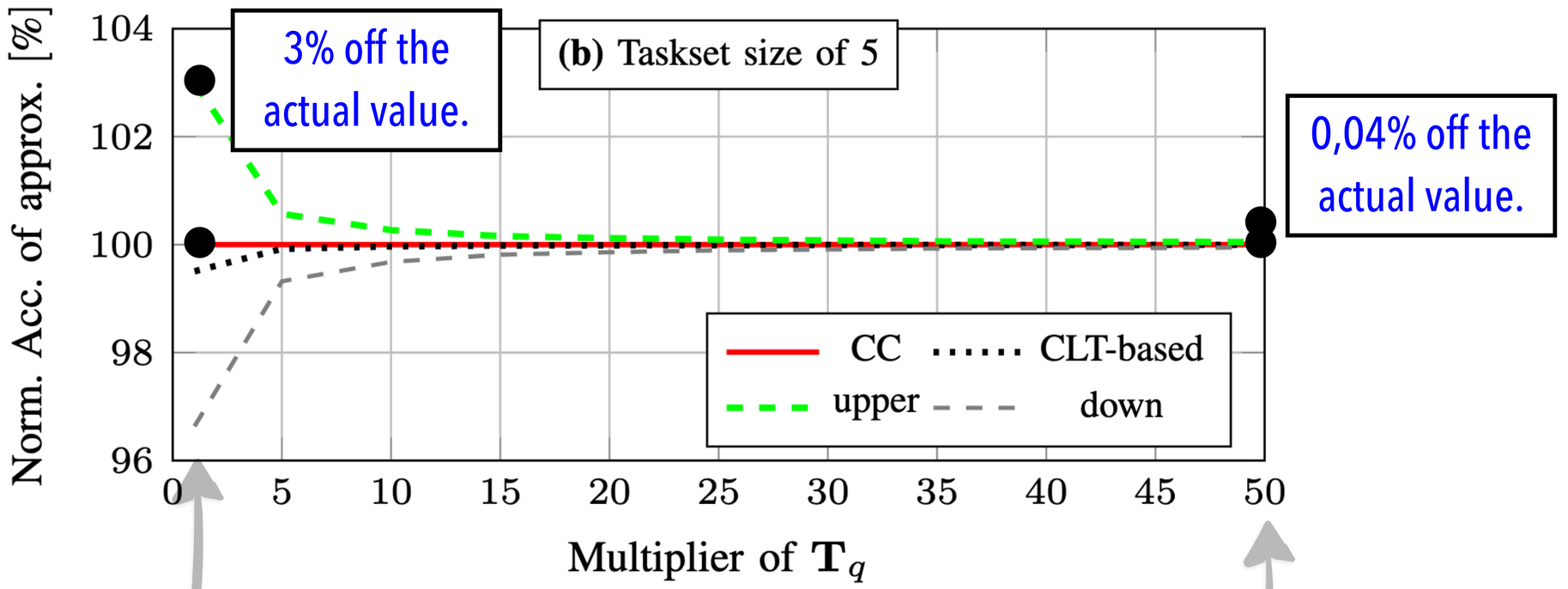
Investigated scenario: Quantile analysis over increasingly longer time intervals.

Variable parameter: Time interval under the analysis, equal to the multiplier of the period of the lowest priority task.

Comparison point 1: Time interval = Period of the lowest-priority task.

Comparison point 2: Time interval = 50 periods of the lowest-priority task.

EVALUATION Approximation Accuracy



*Closer to the red value is better

Investigated scenario: Quantile analysis over increasingly longer time intervals.

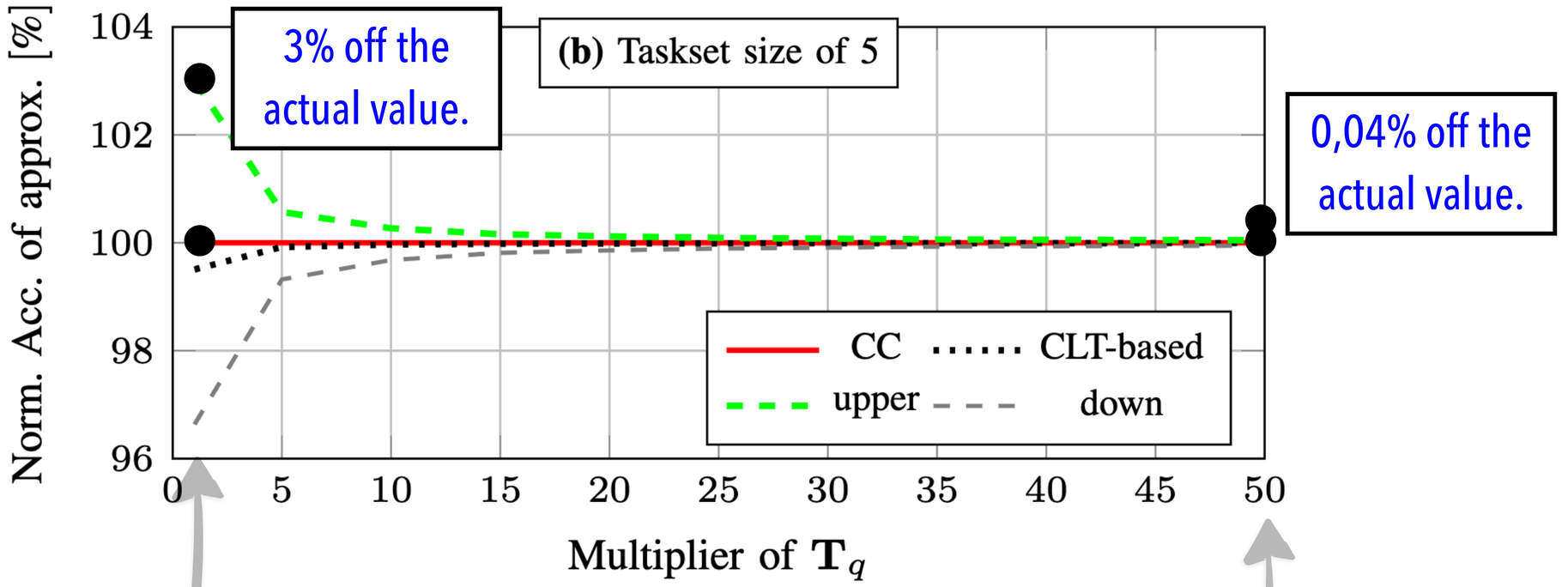
Variable parameter: Time interval under the analysis, equal to the multiplier of the period of the lowest priority task.

Comparison point 1: Time interval = Period of the lowest-priority task.

Comparison point 2: Time interval = 50 periods of the lowest-priority task.

EVALUATION Approximation Accuracy

*Accuracy is **improving** with the increase in the problem size.*



*Closer to the red value is better

Investigated scenario: Quantile analysis over increasingly longer time intervals.

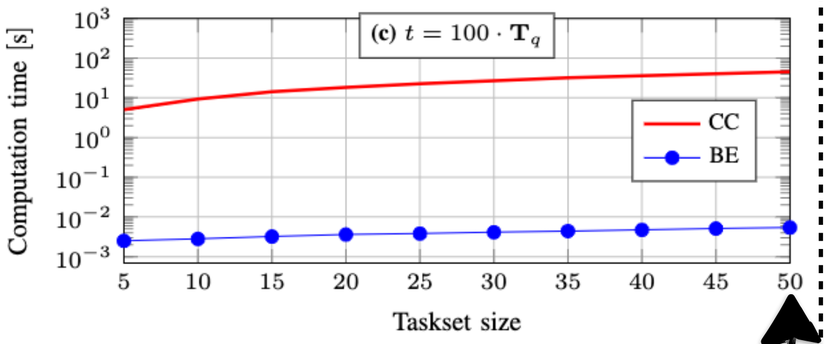
Variable parameter: Time interval under the analysis, equal to the multiplier of the period of the lowest priority task.

Comparison point 1: Time interval = Period of the lowest-priority task.

Comparison point 2: Time interval = 50 periods of the lowest-priority task.

EVALUATION

*What are the benefits of using the **proposed approach** over the most efficient analysis (**circular convolution**) for deriving the **exact distribution**?*



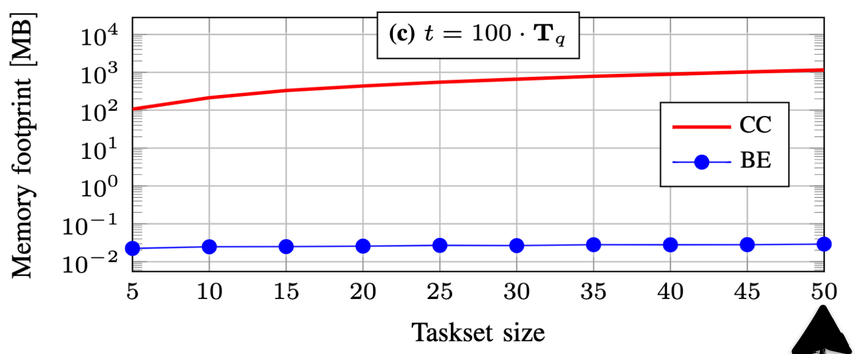
Comparison point: Taskset size = 50.

Circular Convolution (CC): 46 **seconds**

Berry-Essen (BE): 0.005 **seconds**

Computation time is **significantly improved** over the **circular convolution** approach.

Computation Time



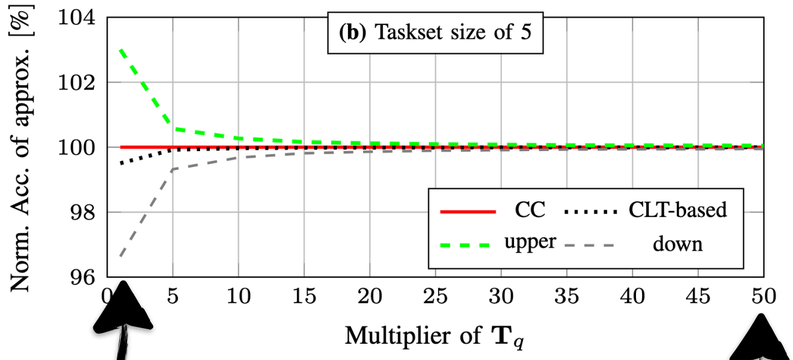
Comparison point: Taskset size = 50.

CC: 1157.8 **MB**

BE: 0.029 **MB**

Memory footprint is **significantly improved** over the **circular convolution** approach.

Memory Footprint



Comparison point 1: one period

BE: 3% off the actual value.

Comparison point 2: 50 periods

BE: 0.04% off the actual value.

Accuracy is **improving** with the increase in the problem size.

Approximation Accuracy

EVALUATION

*What are the benefits of using the **proposed approach** over the most efficient analysis (**circular convolution**) for deriving the **exact distribution**?*

Computation Time

Computation time is **significantly improved** over the **circular convolution** approach.

Memory Footprint

Memory footprint is **significantly improved** over the **circular convolution** approach.

Approximation Accuracy

Accuracy is **improving** with the increase in the problem size.

ACKNOWLEDGMENTS

THANK YOU FOR YOUR ATTENTION